This is a pre-print of a contribution to Proc. PP-RAI 2023. The final version is available online at https://doi.org/10.34658/9788366741928.6.
Full citation:
L. J. Chmielewski, M. Bator and K. Gajowniczek. One-point Hough transform with centred accumulator. In: A. Wojciechowski and P. Lipiński, editors. *Progress in Polish Artificial Intelligence Research 4. Proc. 4th Polish Conference on Artificial Intelligence PP-RAI'2023*, number 2437 in Lodz University of Technology Monographs, pages 49-54, Łódź, Poland, 24-26 Apr 2023. Lodz University of Technology Press.
Last compiled on October 4, 2023.

One-point Hough transform with centred accumulator

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Abstract. *A novel approach for improving the accuracy of the Hough transform by centering the accumulator in the middle of the image is proposed. This improves the results in this crucial image region and optimizes the utilization of the accumulator space. The information on the direction as well as on the sense of edgels is accumulated, which makes it possible to effectively group the edgels into meaningful continuous edges.*

Keywords: *Hough transform, centred accumulator, one-point, directional*

1. Introduction

The method for detecting straight lines in images invented by Paul V. C. Hough in 1959 [3] in the application to analysis of experimental results in high energy physics was not mentioned in the literature for ten years, which could be due to the famous patent filed in 1960 and granted in 1962. The publication by Duda and Hart of a new formulation of the Hough transform (HT) in 1972 [2] has gained extreme popularity. The last survey dedicated solely to HT is probably [6]; further surveys were dedicated to general problems rather than particular methods, e.g. [4]. In 2022 alone the number of papers related to HT was about 13 200. New concepts still emerge, like for example the *Cartesian* HT, with interesting properties, published in 2022 [7]. This was close to the 60-th anniversary of the patent.

The proposition made in this paper is one more modification of HT among many existing solutions. It caters for an improvement of the detection accuracy and of the information content of the results of the Hough transform.

2. Motivation by a practical problem

The study towards extending the informative content of the accumulator in the HT emerged as an answer to the needs in the project on monitoring the behaviour of plants watered daily in a greenhouse. As the measure of the state of water needs in plants their turgor pressure can be used. Its loss in the cells of a plant manifests itself macroscopically with lowering the leaves with respect to the stem. The proper moment of watering is just before the turgor pressure becomes too weak. This phenomenon can be monitored by measuring the changes of angles formed by twigs and leaves and the stem of a plant in a series of images.

The shape primitives suitable for finding angles are straight line elements, possibly merged into longer edges. Images taken in everyday practice in a greenhouse make finding such primitives feasible. The analysis of plants will not be presented in this paper, but images of plants will be used to illustrate the considerations.

It must be stated that the methods other than HT are now frequently used to find lines, circles and ellipses (see e.g. [5]). Robustness, accuracy and lack of training are in our opinion the reasons why the HT is still a method with great potential.

3. Hough transform and its versions

The lines considered in this paper are actually *edges* in which the gradient is known. The edges are found with the directional second derivative zero-crossing detector and the colour image is transformed into greenness intensity by linearly scaling the distance of the H component of the HSV representation from the green hue, from the $\langle 0, 180\rangle$ interval (periodicity considered) into the $\langle 255, 0\rangle$ interval.

Let us consider the geometry shown in Fig. 1. A pixel P_i in which the image intensity gradient is $\vec{G}_i$ lies on a line l_i. Such a pixel is called *edgel*. The line can be defined by the distance R, called radius, between its foot point F_i and the origin of the coordinate system $O(0, 0)$, and the angle φ_i between the x axis and the line OF_i [2] (in fact F_i univocally defines the line [7]; however, accuracies go down as F_i approaches O). The equation of line l going through pixel $P(x, y)$ is

$$x \cos\varphi + y \sin\varphi - R = 0\,. \tag{1}$$

Each edgel $P_i(x_i, y_i)$ with φ_i defined by the direction of its gradient casts a vote in a single point (φ_i, R_i) in the accumulator, where R_i is found from (1) for x_i, y_i, φ_i.

In the classical approach [2], if from (1) it follows that $R < 0$, then $R := -R$ and $\varphi := \lfloor\varphi - \pi\rfloor$, hence $R = |\vec{R}|$ and the sense of edgels is forgotten ($\lfloor\cdot\rfloor$ means here

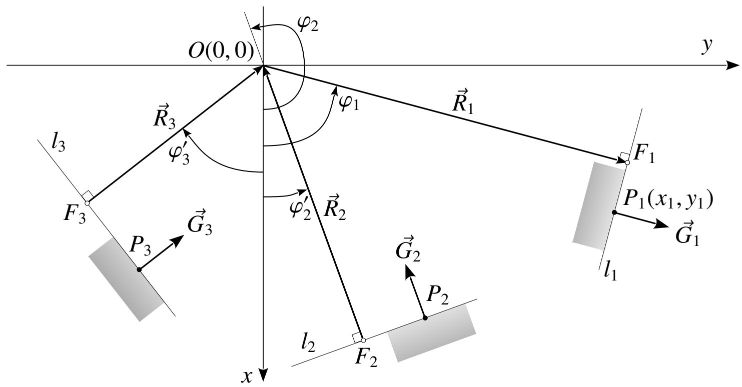

Figure 1. Geometry of the Hough transforms discussed. $P_i(x_i, y_i)$ – pixels of edges which form lines l_i; $\vec{G}_i$ – respective image brightness gradients; $\vec{R}_i$ – HT directed radii in pixels P_i, $\vec{R}_i \parallel \vec{G}_i$; φ_i – HT angles; $\varphi_i' = \varphi_i - \pi$; $i = 1, 2, 3$. F_i – foot of line l_i. Grey rectangles symbolize image intensity gradients.

bringing to the interval $(-\pi, \pi\rangle$). Then, if the image of diagonal D is located in the first quadrant of Oxy, then $\varphi \in (-\pi/2, \pi\rangle$ and $R \in [0, D]$.

The first well known improvement is to extend the range of R to negative values, so that not only the direction, but also the sense of edgels, i.e., of $\vec{G}$, is accumulated. Conventionally, the origin is located in the upper left corner of the image; edges of green objects in Fig. 2a were found in this setting. It can be seen that the density of pixels found as belonging to lines goes down with the distance from the origin. The same appears if the origin is placed in the opposite corner (Fig. 2b).

Therefore, we propose to place the coordinate system origin in the centre of the image. The result can be seen in Fig. 2c. This simple operation seems not to have been proposed until now, to our best knowledge. We shall name this new version of HT the One-Point Hough Transform with Centred Accumulator: OPHT-CA.

Let us consider the contents of the accumulator in these three approaches. An ideal source image for this would be such that there are edgels *everywhere* and in *all* directions; our image of Fig. 3d is its humble approximation. The accumulators for the three HT versions considered are shown in Fig. 3. In the proposed OPHT-CA the volume of the accumulator is utilized the best (no unused black regions).

In the accumulator of PPHT-CA the weak fuzzification with a paraboloidal fuzzifying function is applied [1] (paraboloids can be seen around strong maxima).

It is important that the scanning of the image is done twice: once for accumulation, and a second time for assigning pixels to maxima. A pixel the vote of which

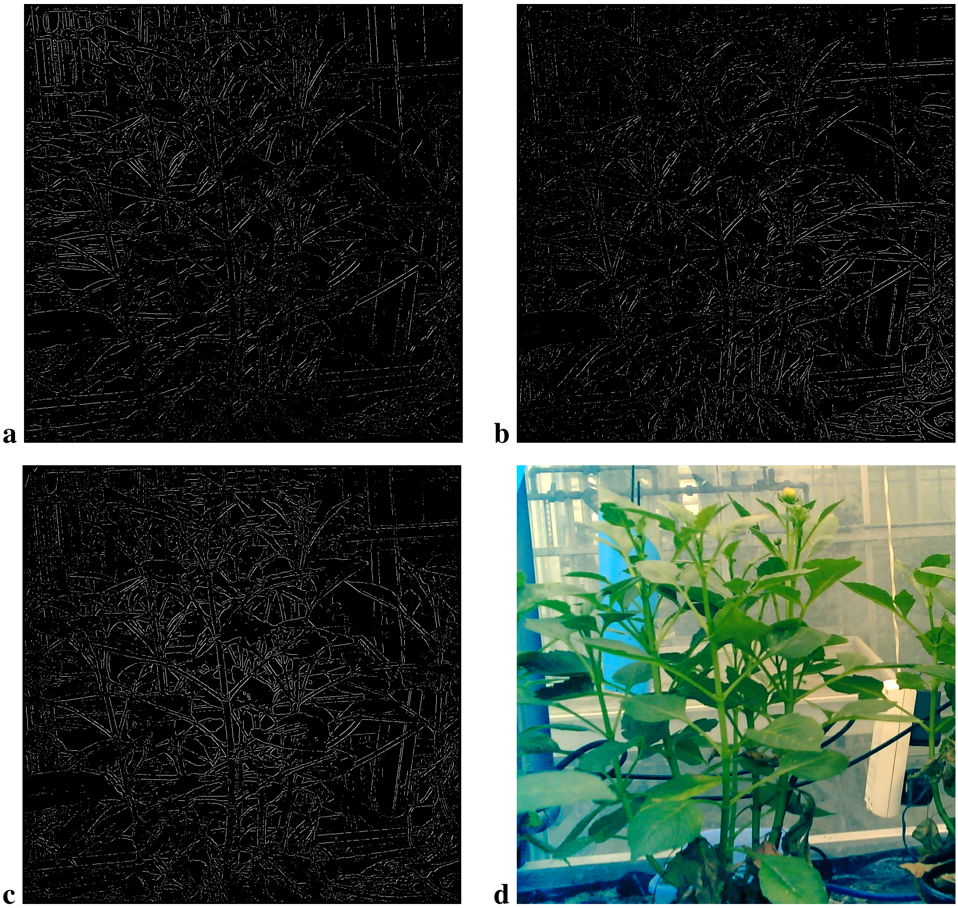

Figure 2. Pixels belonging to lines, origin of the coordinate system $OR\varphi$ located in: (**a**) upper left corner; (**b**) lower right corner; (**c**) centre of the image. (**d**) Source.

falls inside an elliptic neighbourhood of a maximum is assigned the direction related to this maximum, and the intensity being a product of maximum value and edgel gradient modulus. Neighbourhood size corresponds to the scale of the fuzzifying function. A pixel with the vote not assigned to any maximum is dismissed. This reduces the number of potential edgels making edge aggregation effective.

The reduced accuracy of the HT far from the coordinate system origin results from that the inaccuracies related to roundings in the indexing functions (which bind accumulator indices with radius and angle) are growing with the distance

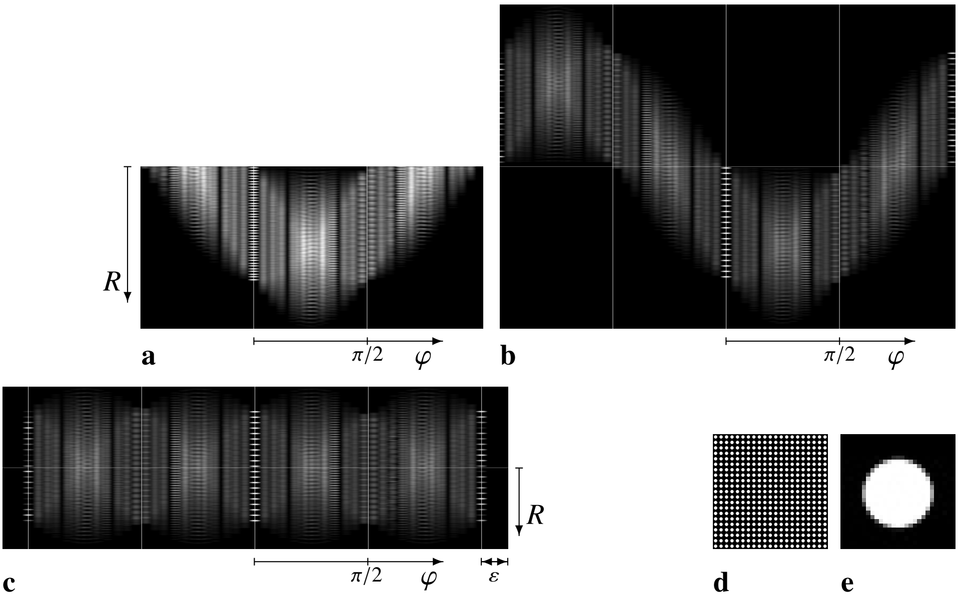

Figure 3. Accumulator in three versions of HT for image **d** with diagonal D. (**a**) Classic version, $R \in [0, D]$, $\varphi \in [-\pi/2, \pi]$; edgel sense forgotten. (**b**) Version with $R \in [-D, D]$, $\varphi \in [-\pi, \pi]$. (**c**) Version with centred origin; $R \in [-D/2, D/2]$, $\varphi \in [-\pi - \varepsilon, \pi + \varepsilon]$, where ε – margin useful in implementation of the accumulator, which is cyclic. (**d**) Source image with diagonal D, formed of 21×21 images (**e**).

from this origin. Discretizing the accumulator more densely to achieve accuracy gain implies the necessity of increasing the scales of the fuzzifying function; both these operations bring longer calculations. So, a compromise between accuracy and time is needed. Accuracy could be tested quantitatively with an image similar to that of Fig. 3d but such tests fall beyond the scope of this short communication. Moving the origin to the image center improves the accuracy at no computational cost, and brings memory reduction by optimizing the use of the accumulator space.

4. Conclusion

A modification of the Hough transform based on its most classic version has been proposed. It consists in placing the centre of the accumulator in the region in which the detection accuracy should be the best, that is, in the centre of the image. This not only improves the accuracy of results, but also leads to optimal use of the accumulator volume. Accumulating the sense of edgels together with their direction, and performing a second scan of the image to assign pixels to maxima in the accumulator, provides for grouping these edgels into contiguous, meaningful edges with a consistent direction. Quantitative analysis of the benefits and costs of these improvements will be carried out in further publications.

It is planned to use the proposed One-Point HT with Centred Accumulator – OPHT-CA – in measurements of plant movements in the process of their watering.

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