

Both Shares in Color Visual Cryptography Can Be Statistically Indistinguishable from Noise

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Keywords

Visual cryptography, Color, Randomness, NIST tests.

Abstract

In visual cryptography, each share must appear statistically indistinguishable from noise. The first share is generated using a pseudorandom number generator; the second one is constructed based on the first one and the image to be encoded. This is true for black-and-white, gray, and color versions of the scheme. In previous work, we proposed a method for generating the second share that preserves essential encoding constraints while aiming to maintain the statistical randomness of the first share. To assess the randomness of shares by applying typical randomness tests (NIST STS) we simulate hundreds of share samples and analyze the resulting p -value distributions. This simulation enables us also to test the uniformity of distributions visually. In this study, we extend our assessment by testing for deviation of these distributions from uniformity using Kolmogorov–Smirnov and chi-squared tests. Results of simulations indicate that our method preserves randomness in the second share to a degree comparable to that of the first one, supporting its use in statistically robust visual encryption of color images.

Introduction

Randomness is a cornerstone of cryptographic security. It underpins not only key generation and protocol unpredictability, but also structural concealment in schemes such as visual cryptography (VC). In VC, a secret image is divided into multiple binary *shares* which reveal the secret image when overlayed and viewed by bare eye. Shares, when viewed individually, must reveal no meaningful information. This concealment depends not merely on logical irreversibility, but on statistical indistinguishability. In fully random cryptography schemes each share must resemble uniform random noise. Other possibilities are that shares simply represent no useful information, or mimic images which contain irrelevant views (Dhiman and Kasana 2018).

In classical schemes, the first share is expected to be random. A physical random process could be used to this aim; however, in practice the random process is simulated with a pseudorandom number generator (PRNG). The second share is computed to two separately desirable targets. First it is of a utmost importance to satisfy the reconstruction constraints to a largest possible extent. Second, the simulation of randomness of the second share is also a target. As a result, while the first share reflects the statistical properties of the PRNG, the second one may contain artifacts introduced by the encoding algorithm. If these artifacts introduce patterns detectable by statistical means, the concealment may be compromised – not visually, but analytically. In previous work, we proposed a method for generating the second share that minimizes such distortion. Our construction modifies the second share in a controlled manner, introducing the information necessary for decoding, while attempting to maintain the original randomness. Informally, the second share should “look just as random” as the first one, even though it is not independently sampled.

Preliminary visual and histogram-based analyses (Chmielewski, Nieniewski, and Orłowski 2022a) suggested that this goal might be achievable: p -values from standard randomness tests (e.g., rank, longest run) applied to sets of shares were broadly well-distributed, with few values falling below conventional significance thresholds. However, such observations, while suggestive, lack formal statistical confirmation (cf. (James 1995)).

In this paper, we address this gap. For a given image, we simulate the process of multiple uses of the algorithm by generating 100 random instances of the first share (with PRNG) and derive corresponding second shares using our method. Each share, treated as a one-dimensional series of bits that represent pixels read by rows or by columns, is tested using a battery of randomness tests (NIST Statistical Test Suite, NIST STS (Bassham, Rukhin, Soto, Nechvatal, Smid, Leigh, Levenson, Vangel, Heckert, and Banks 2010)), and the resulting p -values are aggregated. We analyze these p -value vectors in two ways: (1) by counting how many

fall below a fixed rejection threshold ($\alpha = 0.01$) (as in (Chmielewski, Nieniewski, and Orlowski 2022a)) and (2) by testing their histogram for deviations from uniformity using the Kolmogorov–Smirnov and chi-squared tests. This second-order statistical analysis, i.e., treating p -value distributions as objects of inference, enables a more rigorous evaluation of the method’s ability to preserve randomness. Our findings suggest that the second shares exhibit no statistically significant degradation in randomness relative to the first ones, indicating that a controlled transformation with random as well as deterministic elements need not compromise statistical concealment in visual cryptography.

Surprisingly enough, statistically testing the shares in color visual cryptography was not widely applied. In (Ulutas, Yazici, Nabihev, and Ulutas 2008) shares were tested with the correlation-based tools. Recently, several authors (Toktas, Erkan, and Yetgin 2024; Tang, Lu, Zhang, Huang, Huang, and Wang 2024; Liu and Ding 2024) tested the security of key-based ciphers for images with NIST STS, but their methods had no relation to purely visual cryptography in which bare eye would be enough to reveal the secret image by overlaying the shares.

Visual coding algorithms

In first visual cryptography of black-and-white images each pixel of an image was represented by a 2×2 *tile*. The shares were not random but had a structure which made it possible to represent pixels at two levels: bright (half-white) and dark (black) (Naor and Shamir 1995). For representing images with more shades of grey the dithering techniques were used (see for example (Stinson and Paterson 2018)). For color images, also the dithering schemes with additive as well as subtractive basic colors were used (Yang and Chen 2008; Dhiman and Kasana 2018). We go along the same path, but as the starting point we take an entirely random tile, as far as the random number generator can be considered as actually simulating the randomness correctly. So, the tile is in no way crafted for the coding process.

The R, G, B and K pixels in it are taken from a random integer number generator in $\{0, 1, 2, 3\}$. Throughout this paper 6×6 tiles are used, each representing one color pixel in the coded secret, which is a compromise between the restored image quality and pixel expansion. For each pixel, hence a tile, the information on the numbers of R, G, B and K (black) pixels that need to be represented come from the dithering process: these are the reconstruction constraints. Now, there are two concepts of coding to meet these constraints. We shall concentrate on a single pixel in a pair of tiles, in share 1 and 2. The process is strictly pixelwise parallel.

Coding by hiding: the tiles in the two shares are originally identical, so all the pixels are initially un-

covered. By swapping the pixels, by pairs, in the share 2, with the pairs selected random, covering the unnecessary pixels is attempted, to meet the reconstruction constraints as closely as possible (see (Chmielewski, Nieniewski, and Orlowski 2021) for details).

Coding by unhiding: the tile 2 is formed by choosing for each pixel a color different from that in tile 1, at random, so all the pixels are initially covered (other simple ways to get a random share 2 with all pixels covered exist). By swapping the pixels, by pairs, in the share 2, with the pairs selected at random, uncovering the necessary pixels is attempted to meet the reconstruction constraints as closely as possible (see (Chmielewski, Nieniewski, and Orlowski 2022b) for details).

Let us note that each color, not only black, can hide any other color, due to that overlaid color shares are transparent narrow-band filters.

The pixels in share 2 are only swapped, or randomly generated and swapped, so it is aimed to preserve their statistical randomness while encoding the secret image. In both algorithms errors occur due to that the tiles are random, not engineered for the errorless operation of the algorithms. There are not enough color pixels or some pixels cannot be covered due to lack of enough pixels in another color in share 2. An example of coding and decoding is shown in Figure 1 (fragment of a figure from (Chmielewski, Nieniewski, and Orlowski 2022a), according to licence). In coding by unhiding, the surplus bright pixels can be easily avoided, but the lack of enough color pixels to appear unhidden can occur more frequently than in the coding by unhiding (Chmielewski, Nieniewski, and Orlowski 2022a). For typical data, the coding by hiding is more appropriate, so this method will be considered further in this paper. In Figure 2 an example of decoding a natural image is shown. It can be marginally noticed that the quality attainable with bare eye, for example in field conditions, is very limited, which is typical for this class of decoding methods, while it is possible to greatly enhance the image quality with very simple computations (see (Chmielewski, Nieniewski, and Orlowski 2022a)).

Materials and methods

For each tested image, we generated 100 independent instances of the share 1 using a default pseudorandom number generator. Each corresponding share 2 was computed using the method described above. Both shares were treated as binary data and subjected to statistical randomness evaluation using a standard test suite (NIST STS). For each share, and for each test, we obtained a single p -value indicating the degree to which the observed result is consistent with the null hypothesis of randomness. This produced, per image and per

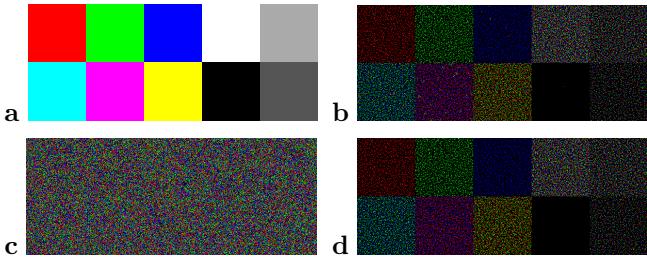


Figure 1: Decoding for a Test Image `test100c`. (a) Original, 100×40 . (b) Decoded 600×240 with the *hiding* method. (c) Share 2, a random image (d) Decoded 600×240 with the *unhiding* method.

test, a vector of 100 p -values for Share 1 and 100 for Share 2, each in two versions: read by rows and read by columns, so there are four sets of data for each tested image. We tested six images frequently used as benchmarks: `baboon` (quarter edge size), `parrots` (half edge size), `peppers`, `Lena`, `Boats` and `test100c`, a small test image used in our previous papers (Chmielewski, Nieniewski, and Orlowski 2022a). There are 15 tests in the NIST STS, some with subtests, which gives 188 tests (default values of parameters were used, unless stated otherwise). In total, there were 4512 independent realizations, or samples, of p -value vectors, 100 elements each vector.

Two levels of analysis were performed on these samples of p -value vectors, each vector constituting a histogram:

Rejection count: the number of p -values falling below a fixed significance threshold ($\alpha = 0.01$). This test (p-v) quantifies how often a given test would flag a sample as non-random under standard criteria.

Uniformity tests: the distribution of p -values was tested for consistency with the uniform distribution $U(0, 1)$ using both the Kolmogorov–Smirnov (K-S) test and Pearson’s chi-squared (χ^2) test. This meta-analysis treats the p -values themselves as data to evaluate second-level randomness.

Results

Across all images and tests, both share 1 and share 2 exhibited low rejection rates, with the number of p -values below 0.01 typically within the expected range under the null hypothesis. In particular, share 2 did not display systematically higher rejection counts than share 1.

Histogram-based uniformity assessments yielded similarly positive outcomes. In the majority of cases, neither the KS test nor the chi-squared test rejected the null hypothesis of uniformity of the p -value histogram at the 0.05 level. Where rejections did occur, they were scattered and statistically compatible with type I error rates. Figure 3 (fragment of a figure from (Chmielewski, Nieniewski, and Orlowski 2022a), according to licence)

presents representative p -value histograms for a single test (Rank) across 100 share 1 and share 2 instances, each by rows and by columns, illustrating their visual similarity. More histograms, also for a method without the randomness property, can be seen in (Chmielewski, Nieniewski, and Orlowski 2022a).

Tables 1, 2 and 3 summarize p-v, K-S and χ^2 tests for some of the many possible image-test-share-direction combinations, confirming the absence of systematic rejections and their generally marginal counts among the 4512 samples of the p -value histograms.

Not all the tests revealed failures. Number of non-randomness cases detected by NIST tests are shown in Table 4 for both methods (test names were abbreviated to reduce table width). The `OverlappingTemplate` opens the ranking for both methods. Other tests revealed much less failures. It can be observed that some tests detected no failures at all; these were: `ApproximateEntropy`, `CumulativeSums 2`, `LinearComplexity`, `Rank`, `Runs`, `Universal`, and some subtests of `NonOverlappingTemplate`, `RandomExcursions` and `RandomExcursionsVariant`.

These results indicate that the mixed random and deterministic transformations applied to generate share 2 do not introduce detectable structure that would be revealed by typical randomness tests. In particular, the p -value distributions of share 2 are statistically indistinguishable from those of share 1, both in rejection frequency and in second-order uniformity.

Discussion

The primary aim of this study was to assess whether the transformation used to encode information in the second share of a visual cryptographic pair necessarily degrades the statistical appearance of randomness.

Our findings suggest that it does not. Across a wide range of images and tests, the second shares generated using our method passed standard randomness assessments at rates comparable to purely random first shares. Neither rejection counts nor p -value distributions indicated systematic anomalies.

These results provide evidence that a carefully designed mixed random and deterministic transformation can preserve the statistical noise characteristics essential for

Table 1: Rejections by Test

Test	Share 1	Share 2	Rows	Cols	Fails/Samps
<i>Hiding</i> method					
p-v	43	36	45	34	79/4512
K-S	16	29	16	29	46/4512
χ^2	19	27	23	23	46/4512
<i>Unhiding</i> method					
p-v	49	51	49	51	100/4512
K-S	16	20	21	15	36/4512
χ^2	24	25	28	21	49/4512



Figure 2: Decoding for a Natural Image *boats*. (a) Original, 787×576 . (b) Decoded 4722×3456 with the *hiding* method. (c) Share 2, upper left quarter, a random image

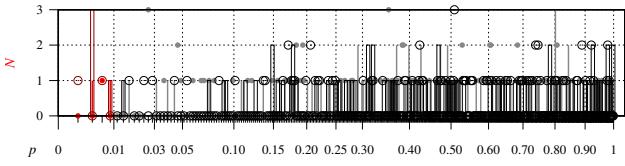


Figure 3: Histograms of 400 p -values for Test *Rank*, Image *boats*, *Hiding* Method. Marks: bright full: share 1, dark empty: share 2, circles: by rows, bars: by columns; red: 9 rejections total

Table 2: Rejections in Test Combinations

Test	Counts/4512 Samples		
<i>Hiding</i> method			
Single	p -v: 79	KS: 41	χ^2 : 46
Pairs	p -v, K-S: 8	p -v, χ^2 : 9	K-S, χ^2 : 14
3-tuple		p -v, K-S, χ^2 : 7	
<i>Unhiding</i> method			
Single	p -v: 100	KS: 36	χ^2 : 49
Pairs	p -v, K-S: 9	p -v, χ^2 : 9	K-S, χ^2 : 13
3-tuple		p -v, K-S, χ^2 : 7	

Table 3: Rejections by Shares/Directions

Case	Counts/4512 Samples		
<i>Hiding</i> method			
By shares	Share 1: 69	Share 2: 73	Both: 8
By directions	Rows: 74	Columns: 68	Both: 6
<i>Unhiding</i> method			
By shares	Share 1: 77	Share 2: 84	Both: 7
By directions	Rows: 83	Columns: 78	Both: 9

the security of visual cryptography. Importantly, this holds not only at the level of individual tests, but also in aggregate behavior: the p -value distributions themselves remain uniform, suggesting that the transformation does not bias or constrain the randomness landscape.

From a methodological standpoint, this study demonstrates the utility of meta-level randomness evaluation, i.e., treating p -value vectors as analyzable objects. This approach adds a layer of robustness beyond pass/fail summaries, enabling the detection of subtle statistical distortions that may not manifest as outright rejections. Finally, the use of two independent statistical environments (Python and R) for validation strengthens the reliability of the results and minimizes the risk of software-related bias.

Conclusion

In this paper, we analyzed whether the generation of the second share in visual cryptography compromises its statistical randomness. Using standard randomness tests and second-order evaluations of p -value distributions, generated in a simulated experiment, we found no evidence of such degradation. Our method for constructing the second share produces outputs that, despite being algorithmically constrained, remain statistically indistinguishable from noise. This confirms its suitability for secure visual encryption and demonstrates that controlled determinism can coexist with apparent randomness. More broadly, our work shows that meta-analysis of test outputs provides a powerful tool for validating the integrity of cryptographic structures under transformation.

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Table 4: Numbers of Failures (N) Revealed by NIST Tests

Hiding method		Unhiding method			
#	N	Test name, subtest	#	N	Test name, subtest
1	12	OverlapTemp1	1	14	OverlapTemp1
2	3	FFT	2	4	NonOvrlTemp1 22
3	3	NonOvrlTemp1 1	3	4	NonOvrlTemp1 26
4	3	NonOvrlTemp1 26	4	4	RandExcurs 8
5	3	NonOvrlTemp1 30	5	3	NonOvrlTemp1 67
6	3	NonOvrlTemp1 73	6	3	NonOvrlTemp1 73
7	3	NonOvrlTemp1 75	7	3	NonOvrlTemp1 79
8	3	NonOvrlTemp1 135	8	3	NonOvrlTemp1 81
9	2	BlockFreq	9	3	NonOvrlTemp1 86
10	2	NonOvrlTemp1 2	10	3	NonOvrlTemp1 111
11	2	NonOvrlTemp1 3	11	2	Frequency
12	2	NonOvrlTemp1 12	12	2	FFT
13	2	NonOvrlTemp1 13	13	2	NonOvrlTemp1 25
14	2	NonOvrlTemp1 15	14	2	NonOvrlTemp1 29
15	2	NonOvrlTemp1 25	15	2	NonOvrlTemp1 31
16	2	NonOvrlTemp1 32	16	2	NonOvrlTemp1 33
17	2	NonOvrlTemp1 40	17	2	NonOvrlTemp1 34
18	2	NonOvrlTemp1 45	18	2	NonOvrlTemp1 35
19	2	NonOvrlTemp1 48	19	2	NonOvrlTemp1 42
20	2	NonOvrlTemp1 71	20	2	NonOvrlTemp1 66
21	2	NonOvrlTemp1 76	21	2	NonOvrlTemp1 84
22	2	NonOvrlTemp1 81	22	2	NonOvrlTemp1 85
23	2	NonOvrlTemp1 90	23	2	NonOvrlTemp1 87
24	2	NonOvrlTemp1 95	24	2	NonOvrlTemp1 95
25	2	NonOvrlTemp1 109	25	2	NonOvrlTemp1 97
26	2	NonOvrlTemp1 110	26	2	NonOvrlTemp1 102
27	2	NonOvrlTemp1 119	27	2	NonOvrlTemp1 103
28	2	NonOvrlTemp1 131	28	2	NonOvrlTemp1 106
29	2	NonOvrlTemp1 136	29	2	NonOvrlTemp1 117
30	2	NonOvrlTemp1 137	30	2	NonOvrlTemp1 135
31	2	RandExcurs 5	31	2	NonOvrlTemp1 147
32	2	RandExcurs 8	32	2	RandExcVar 2
33	2	RandExcVar 5	33	2	RandExcVar 4
34	2	RandExcVar 6	34	2	RandExcVar 18
35	1	CumulatSums 1	35	2	Serial 2
36	1	LongestRun	36	1	NonOvrlTemp1 1
37	1	Rank	37	1	NonOvrlTemp1 3
38	1	NonOvrlTemp1 5	38	1	NonOvrlTemp1 6
39	1	NonOvrlTemp1 7	39	1	NonOvrlTemp1 11
40	1	NonOvrlTemp1 9	40	1	NonOvrlTemp1 13
41	1	NonOvrlTemp1 10	41	1	NonOvrlTemp1 14
42	1	NonOvrlTemp1 11	42	1	NonOvrlTemp1 18
43	1	NonOvrlTemp1 18	43	1	NonOvrlTemp1 19
44	1	NonOvrlTemp1 20	44	1	NonOvrlTemp1 21
45	1	NonOvrlTemp1 31	45	1	NonOvrlTemp1 24
46	1	NonOvrlTemp1 36	46	1	NonOvrlTemp1 27
47	1	NonOvrlTemp1 37	47	1	NonOvrlTemp1 28
48	1	NonOvrlTemp1 38	48	1	NonOvrlTemp1 40
49	1	NonOvrlTemp1 39	49	1	NonOvrlTemp1 41
50	1	NonOvrlTemp1 43	50	1	NonOvrlTemp1 48
51	1	NonOvrlTemp1 51	51	1	NonOvrlTemp1 49
52	1	NonOvrlTemp1 52	52	1	NonOvrlTemp1 50
53	1	NonOvrlTemp1 56	53	1	NonOvrlTemp1 51
54	1	NonOvrlTemp1 58	54	1	NonOvrlTemp1 52
55	1	NonOvrlTemp1 60	55	1	NonOvrlTemp1 55
56	1	NonOvrlTemp1 65	56	1	NonOvrlTemp1 56
57	1	NonOvrlTemp1 66	57	1	NonOvrlTemp1 57
58	1	NonOvrlTemp1 77	58	1	NonOvrlTemp1 58
59	1	NonOvrlTemp1 82	59	1	NonOvrlTemp1 59
60	1	NonOvrlTemp1 83	60	1	NonOvrlTemp1 60
61	1	NonOvrlTemp1 84	61	1	NonOvrlTemp1 61
62	1	NonOvrlTemp1 85	62	1	NonOvrlTemp1 62
63	1	NonOvrlTemp1 87	63	1	NonOvrlTemp1 74
64	1	NonOvrlTemp1 88	64	1	NonOvrlTemp1 75
65	1	NonOvrlTemp1 91	65	1	NonOvrlTemp1 76
66	1	NonOvrlTemp1 92	66	1	NonOvrlTemp1 77
67	1	NonOvrlTemp1 93	67	1	NonOvrlTemp1 80
68	1	NonOvrlTemp1 94	68	1	NonOvrlTemp1 82
69	1	NonOvrlTemp1 96	69	1	NonOvrlTemp1 83
70	1	NonOvrlTemp1 97	70	1	NonOvrlTemp1 93
71	1	NonOvrlTemp1 98	71	1	NonOvrlTemp1 96
72	1	NonOvrlTemp1 102	72	1	NonOvrlTemp1 99
73	1	NonOvrlTemp1 107	73	1	NonOvrlTemp1 101
74	1	NonOvrlTemp1 111	74	1	NonOvrlTemp1 104
75	1	NonOvrlTemp1 114	75	1	NonOvrlTemp1 107
76	1	NonOvrlTemp1 115	76	1	NonOvrlTemp1 109
77	1	NonOvrlTemp1 116	77	1	NonOvrlTemp1 113
78	1	NonOvrlTemp1 117	78	1	NonOvrlTemp1 114
79	1	NonOvrlTemp1 120	79	1	NonOvrlTemp1 116
80	1	NonOvrlTemp1 123	80	1	NonOvrlTemp1 118
81	1	NonOvrlTemp1 127	81	1	NonOvrlTemp1 120
82	1	NonOvrlTemp1 134	82	1	NonOvrlTemp1 122
83	1	NonOvrlTemp1 138	83	1	NonOvrlTemp1 126
84	1	NonOvrlTemp1 142	84	1	NonOvrlTemp1 127
85	1	NonOvrlTemp1 146	85	1	NonOvrlTemp1 128
86	1	RandExcurs 1	86	1	NonOvrlTemp1 130
87	1	RandExcVar 1	87	1	NonOvrlTemp1 131
88	1	RandExcVar 3	88	1	NonOvrlTemp1 132
89	1	RandExcVar 4	89	1	NonOvrlTemp1 134
90	1	RandExcVar 10	90	1	NonOvrlTemp1 138
91	1	RandExcVar 11	91	1	NonOvrlTemp1 141
92	1	RandExcVar 13	92	1	NonOvrlTemp1 143
93	1	RandExcVar 14	93	1	NonOvrlTemp1 145
94	1	RandExcVar 15	94	1	NonOvrlTemp1 148
			95	1	RandExcurs 7
			96	1	RandExcVar 3
			97	1	RandExcVar 5
			98	1	RandExcVar 8
			99	1	RandExcVar 10
			100	1	RandExcVar 12
			101	1	RandExcVar 15
			102	1	Serial 1

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