Both Shares in Color Visual Cryptography Can Be Statistically Indistinguishable from Noise

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Contents

Cryptography and steganography

- Cryptography has been the subject of interest at least for several centuries
- Steganography, its branch: hiding a secret message in another, overt message
- Overt message is intended to hide the secret message, but it also distracts the third parties from the fact that something is hidden
- Even if the coding is unbreakable, it is still susceptible to the simplest attack – the cutoff of the transmission channel
- Therefore, hiding not only the secret, but also the fact that a transmission takes place, is of great value
- Transmitting a structured but unreadable message evokes interest of the third parties
- Our world is full of noise, so noise does not attract too much attention
- A concept of coding a color image in pure noise will be presented
- In the literature, little or no attention was paid to true randomness of shares; usually shares were described as *looking random*

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Classic Visual Cryptography (VC)

Secret image

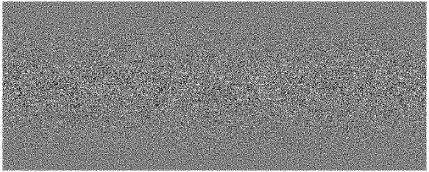
Details will be presented further

Concept: (Naor and Shamir 1995; Naor and Shamir 1997). Images: (Orłowski and Chmielewski 2019a).

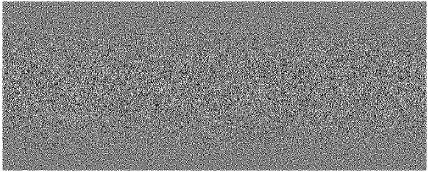
Classic Visual Cryptography (VC)

Secret image

↓ coding in two *shares*



+



Neither *share* contains information on the secret, but their structure is specific

Details will be presented further

Concept: (Naor and Shamir 1995; Naor and Shamir 1997). Images: (Orłowski and Chmielewski 2019a).

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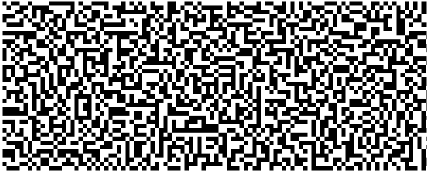
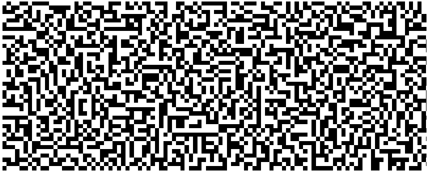


+



Neither *share* contains information on the secret, but their structure is specific (UL corn. $\times 10$)

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Classic Visual Cryptography (VC)

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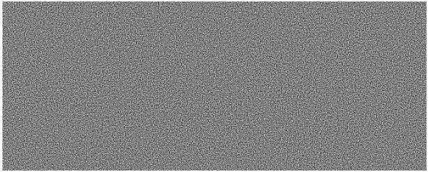

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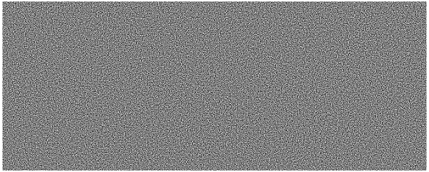

↑ decoding by simply overlaying

Image decoded with unarmed eye

Neither *share* contains information on the secret, but their structure is specific

Details will be presented further

Concept: (Naor and Shamir 1995; Naor and Shamir 1997). Images: (Orłowski and Chmielewski 2019a).

Meaningful shares

One of the concepts of distracting the third party from the coding process is *meaningful shares*

Secret image

Advanced mathematical theory, high quality color methods and multi-participant schemes exist

Concept, images: (Ateniese, Blundo, Santis, and Stinson 2001). Extensions: see e.g. (Dhiman and Kasana 2018).

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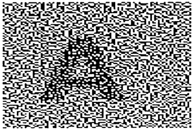
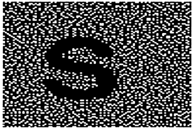
Secret image

↓ coding

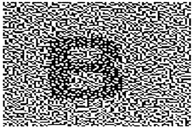


+

Decoded image



↑ decoding



Shares contain images irrelevant to the secret; their structure is still specific

Advanced mathematical theory, high quality color methods and multi-participant schemes exist

Concept, images: (Ateniese, Blundo, Santis, and Stinson 2001). Extensions: see e.g. (Dhiman and Kasana 2018).

Random shares in black-and-white

Secret image

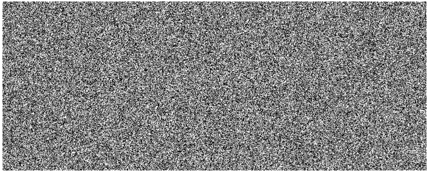
Details will be presented further

B-W: (Orłowski and Chmielewski 2019b). Color: (Chmielewski, Nieniewski, and Orłowski 2021a), CORES 2021.

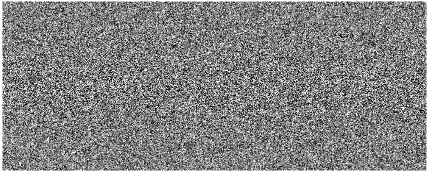
Random shares in black-and-white

Secret image

↓ coding



+



Neither *share* contains information on the secret; their contents is random

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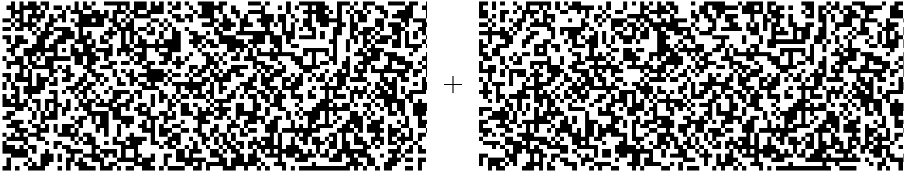

Neither *share* contains information on the secret; their contents is random (UL corn. ×10)

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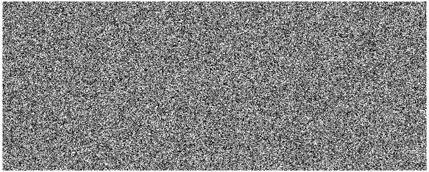

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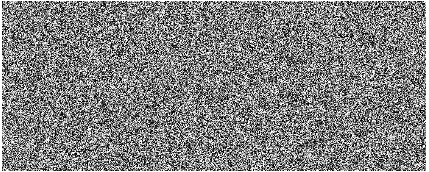

↑ decoding

Decoded, with inevitable errors

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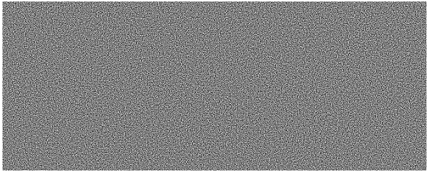
B-W: (Orłowski and Chmielewski 2019b). Color: (Chmielewski, Nieniewski, and Orłowski 2021a), CORES 2021.

Errors caused by randomness

Decoded classically, no errors

↑ decoding



Decoded randomly, with errors

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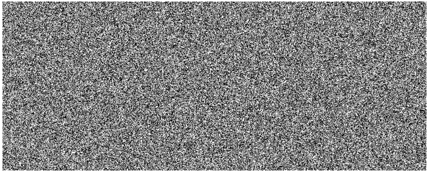

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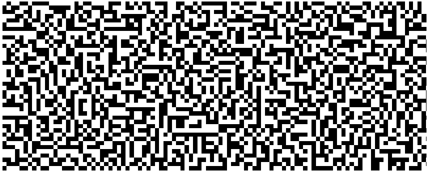

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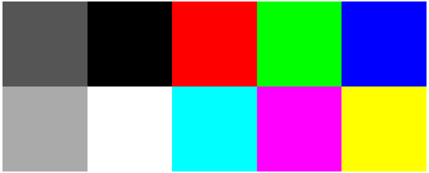
B-W: (Orłowski and Chmielewski 2019b). Color: (Chmielewski, Nieniewski, and Orłowski 2021a), CORES 2021.

Random shares in color

Secret image

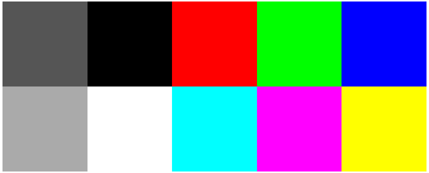


Details will be presented further

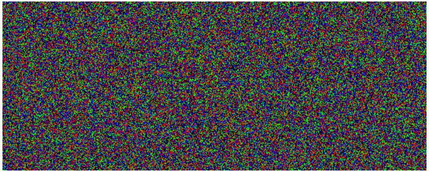
CORES '21 (Chmielewski, Nieniewski, and Orłowski 2021a)

Random shares in color

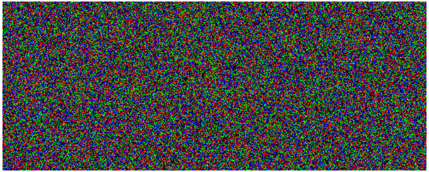
Secret image



↓ coding



+



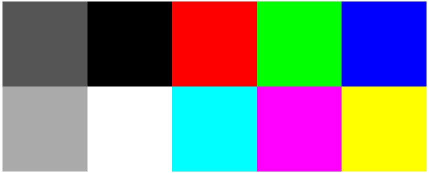
Shares have random contents made of R, G, B, K pixels

Details will be presented further

CORES '21 (Chmielewski, Nieniewski, and Orłowski 2021a)

Random shares in color

Secret image



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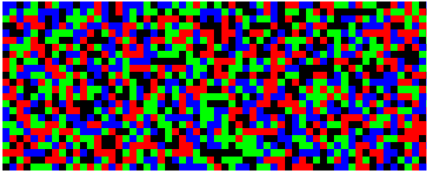

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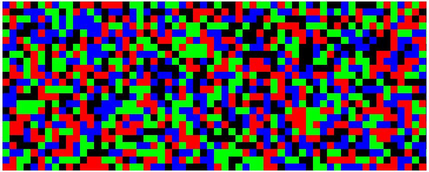

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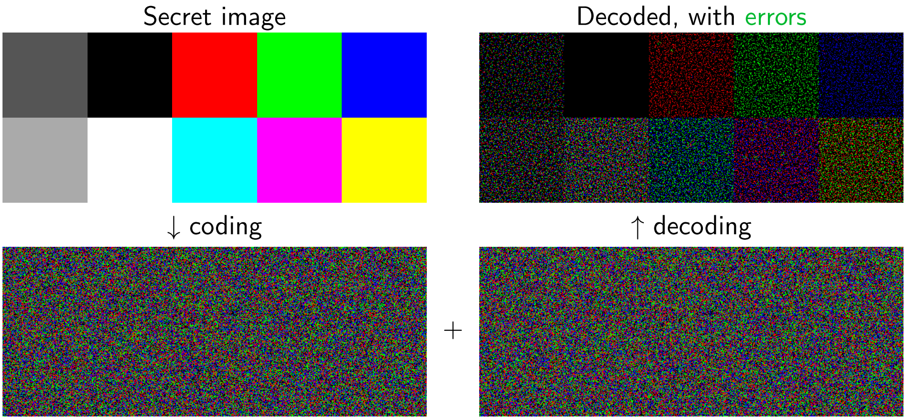

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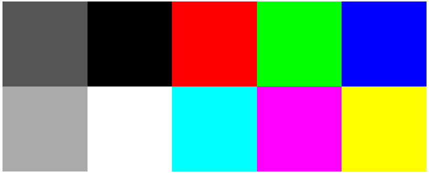

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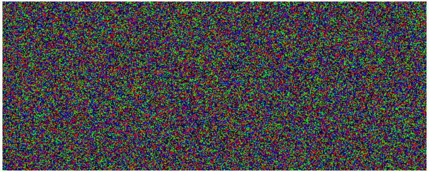
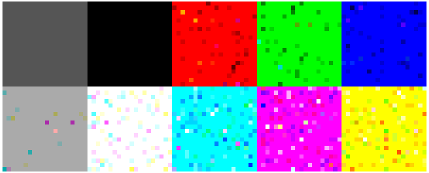
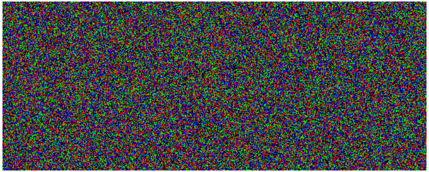


Can be numerically restored easily



↑ decoding



+

Shares have random contents made of R, G, B, K pixels

Details will be presented further

CORES '21 (Chmielewski, Nieniewski, and Orłowski 2021a)

Main content

- Classical coding of black-and-white images
- Coding of black-and-white images in random shares
- Randomness at a cost of errors
- Going to color: Coding of color images in random shares
- Two methods of coding: by hiding and by unhiding
- Testing the randomness – a simulated random experiment
- Histograms of p-values – small number of failures, histogram flatness
- Results

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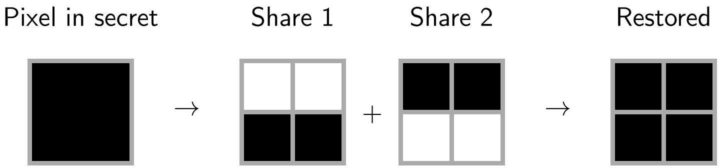
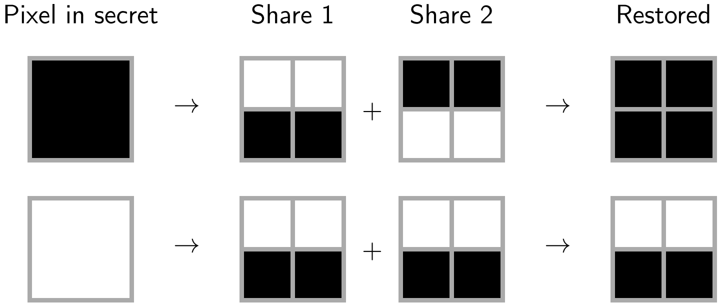
Classic visual cryptography explained



Source: (Naor and Shamir 1995; Naor and Shamir 1997).

Classic visual cryptography explained



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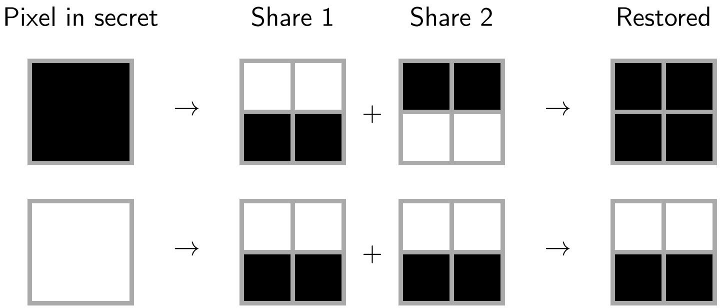


Share 1: drawn at random from *tiles* below. Share 2: chosen according to share 1 and secret.

Source: (Naor and Shamir 1995; Naor and Shamir 1997).

Classic visual cryptography – now we understand it

Secret image

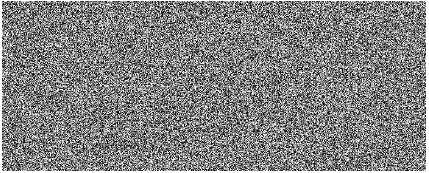
Share 1 is random within limits; share 2 is random similarly, and not correlated with secret.

Concept: (Naor and Shamir 1995; Naor and Shamir 1997). Images: (Orłowski and Chmielewski 2019a).

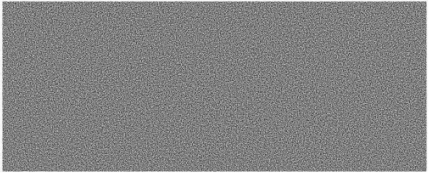
Classic visual cryptography – now we understand it

Secret image

↓ coding in two *shares*



+



Neither *share* contains information on the secret, but their structure is specific.

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Classic visual cryptography – now we understand it

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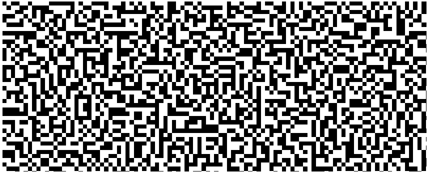
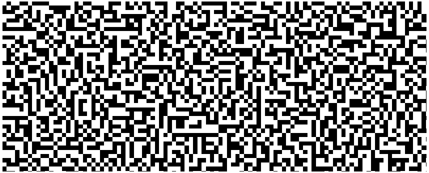


+



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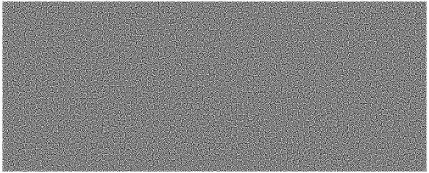
Classic visual cryptography – now we understand it

Secret image

Image decoded, 2×2 larger

↓ coding in two *shares*

↑ decoding by simply overlaying



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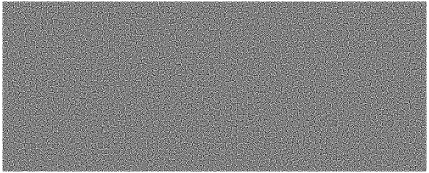

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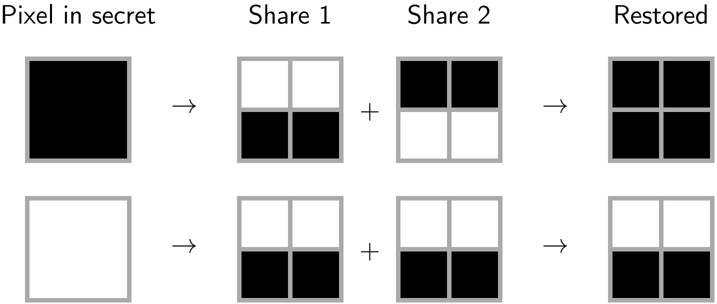
Random B-W visual cryptography explained

Classic: only tiles with 2 pixles white, 2 pixels black, for accurate coding:

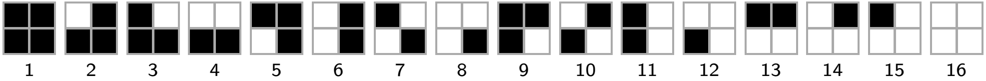
Share 1: drawn at random from tiles below. Share 2: chosen according to share 1 and secret.

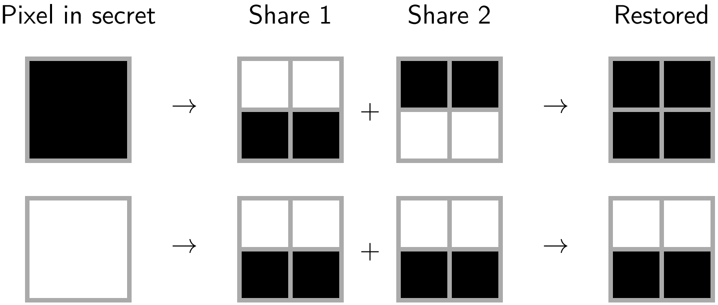


Random B-W visual cryptography explained

Random: with all possible tiles – errors possible:

Share 1: drawn at random from tiles below. Share 2: chosen according to share 1 and secret.





we were lucky

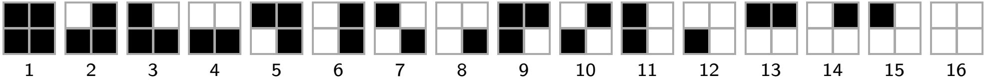
no errors

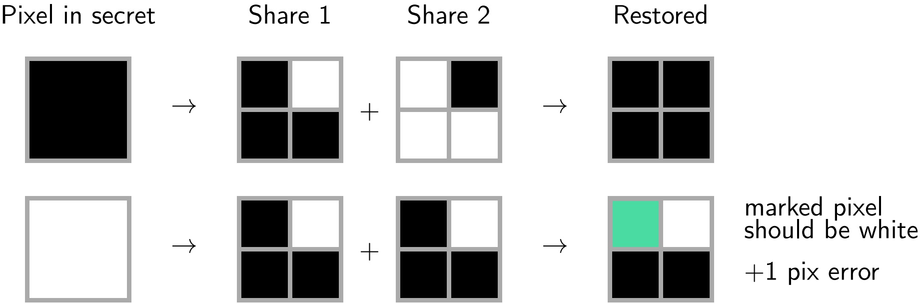
Source: (Orłowski and Chmielewski 2019a; Orłowski and Chmielewski 2019c).

Random B-W visual cryptography explained

Random: with all possible tiles – errors possible:

Share 1: drawn at random from tiles below. Share 2: chosen according to share 1 and secret.





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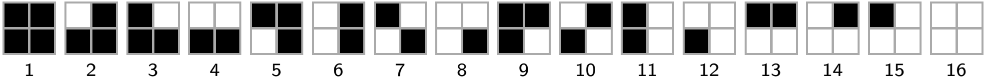

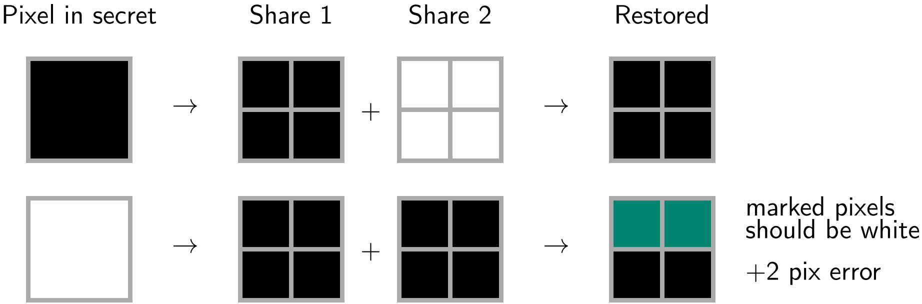

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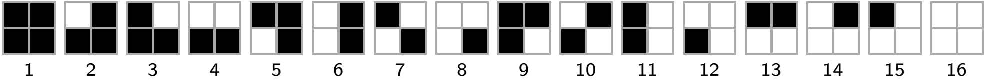
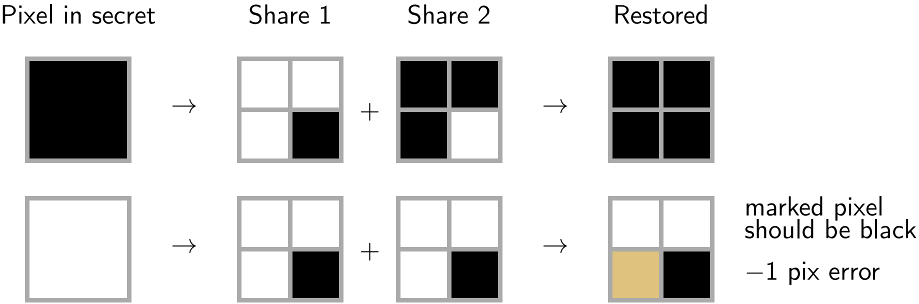


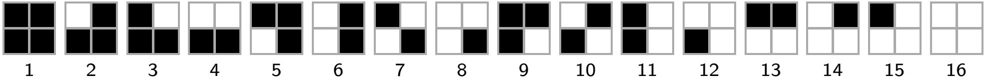


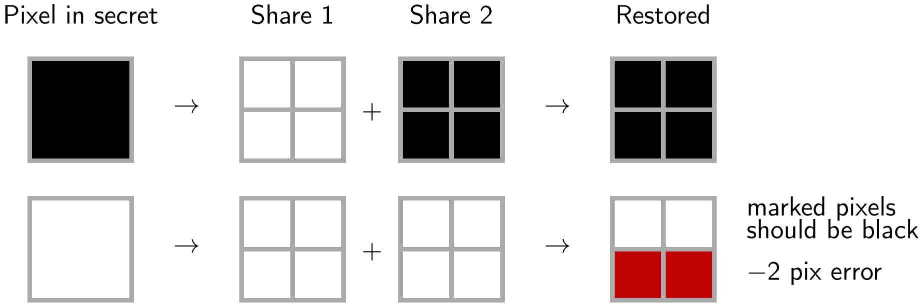
Source: (Orłowski and Chmielewski 2019a; Orłowski and Chmielewski 2019c).

Random B-W visual cryptography explained

Random: with all possible tiles – errors possible:

Share 1: drawn at random from tiles below. Share 2: chosen according to share 1 and secret.





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Random B-W visual cryptography: errors

Decoded classically

Decoded randomly, with errors

Random B-W visual cryptography: errors

Decoded classically

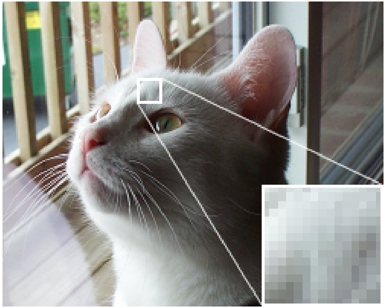
Decoded randomly, with errors

Errors: +1 pix, +2 pix, −1 pix, −2 pix

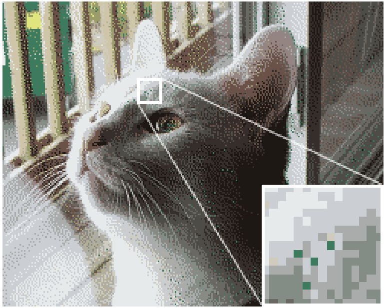
Introducing color

Dithering and color quantization

Original, full color



Dithered, 16-color palette



- Palettes with R, G, B and K (black) pixels only
- Transparencies treated as light emitting device, hence additive color model
- Dithering and color quantization is not in scope of this presentation

Source: (Wikipedia contributors 2021).

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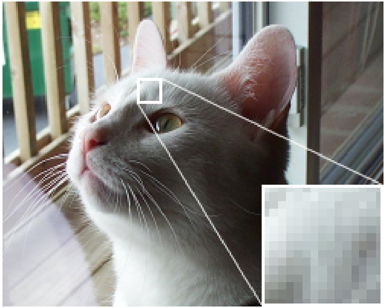

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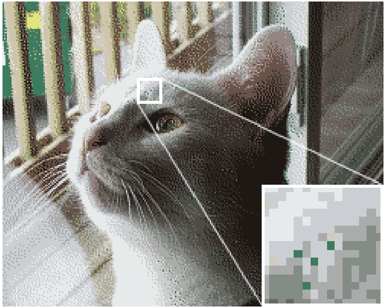

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Introducing segments

- Each pixel of the secret was represented with a *tile*, 2×2
- Now it will be represented with a *segment* consisting of *tiles*
- Segment is large enough to accommodate color: 3×3 tiles.

Source: (Chmielewski, Gawdzik, and Orłowski 2019).

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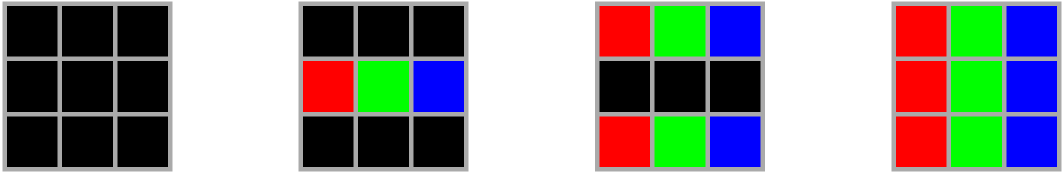

black | 1/3 grey | 2/3 grey | white

Source: (Chmielewski, Gawdzik, and Orłowski 2019).

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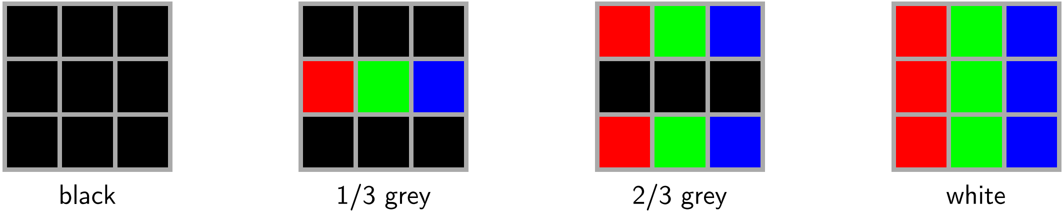

- R, G, B, C=G+B, M=R+B, Y=R+G and W=R+G+B at four levels can be represented

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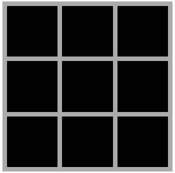
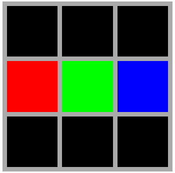
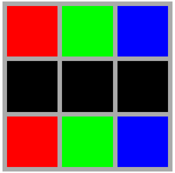


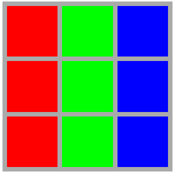
black



1/3 grey



2/3 grey



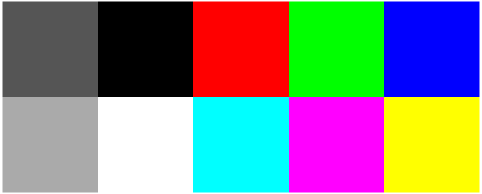
white

- R, G, B, C=G+B, M=R+B, Y=R+G and W=R+G+B at four levels can be represented
- $4^3 = 64$ color palette
- From dithering $\rightarrow$ numbers $\in \{0, 1, 2, 3\}$ of pixels necessary in R, G, B

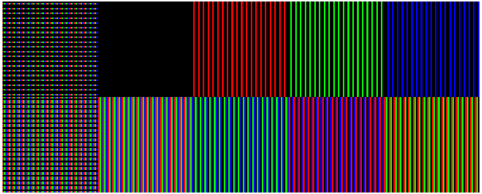
Source: (Chmielewski, Gawdzik, and Orłowski 2019).

Dithering and decomposing into color stripes

Original, full color



Dithered and decomposed



- 64-color palette is not bad, but decomposition into color stripes brings quality loss
- Decomposition is necessary – it makes it possible to represent color by R, G and B

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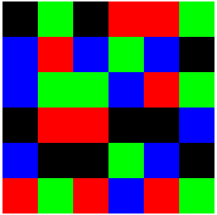
A totally random color tile



Each tile is formed of random pixels R, G, B, K.

BTW, it does not have to be 6×6 (not used here)

Source: (Chmielewski, Nieniewski, and Orłowski 2021a).

- Therefore, the first share is (pseudo)random by definition – it is formed by drawing values $\{1, 2, 3, 4\} \equiv \{R, G, B, K\}$ from a (pseudo)random number generator
- The second share is formed from the first one, but it is modified to code the secret image
- Qn: does this modification make the second share "less random"?
- Randomness in discrete sets means *no structure can be detected*
- Structure $\rightarrow$ not random; otherwise $\rightarrow$ no evidence of randomness
- Randomness cannot be *detected*; its lack can, so as many tests as possible are needed
- We shall return to this after considering the problem of errors

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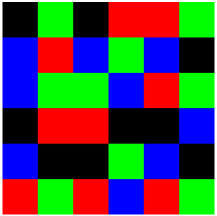

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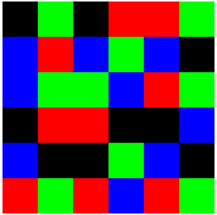

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- Now, segment 1 contains randomly drawn R, G, B and K pixels – **randomness!**
- Only operation on a segment in share 2 is swapping the pixels: **randomness** hopefully maintained

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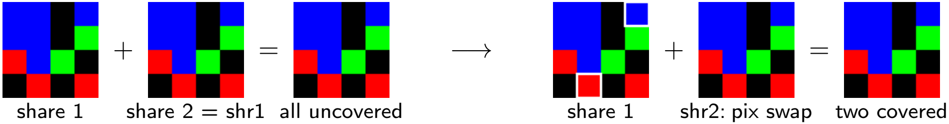

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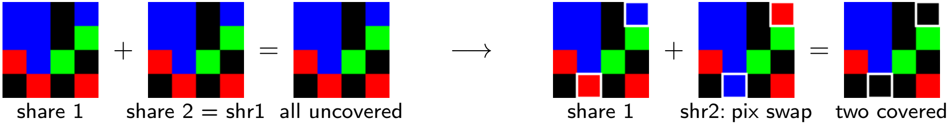

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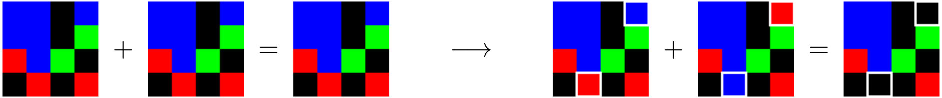

share 1 + share 2 = shr1 = all uncovered → share 1 + shr2: pix swap = two covered

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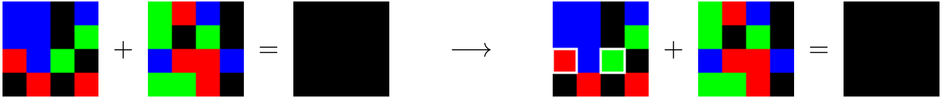
***Unhiding* method: initially shares opposite (covering) → unhiding by swapping**



share 1 + opposite to shr1 = all covered → share 1 + shr2: pix swap = one uncovered

Surely no pixels can remain unhidden

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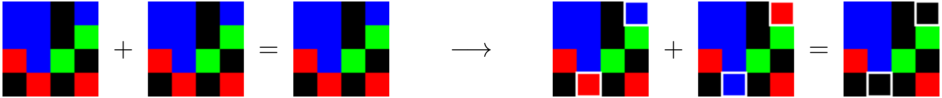

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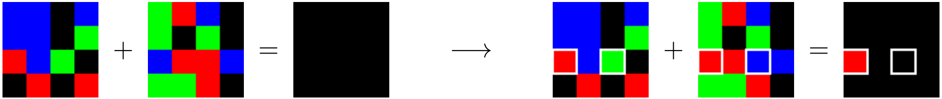

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Algorithm – Truly random coding in one segment

Algorithm (simplified):

- **Input:** random `seg1 = seg2` (***hiding***) or `seg1` $\not\equiv$ `seg2` (***unhiding***)
  Input: numbers of R, G, B pixels `planned to be color` according to dithering
- For each segment in the image:
  - **While** planned (from dithering) number of uncovered color pixels not attained and there are pixels to choose from
    1. Choose 2 pixels `pix1` and `pix2` at random
    2. **If** `pix1` or `pix2` is `fixed` **then** goto 1
    3. **If** swapping `pix1` with `pix2` in `seg2` would `be profitable` **then**
       // condition `be profitable` means:
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       Swap `pix1` with `pix2` in `seg2`
- **Output:** seg2 with only `planned` pixels uncovered, if possible

Comments:

- Randomness is the goal, not efficiency, hence minimum protection against repetitions
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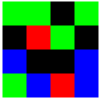
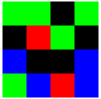
We need **4 R** pixels, but random shares are  +  $\Longrightarrow$ No way, **missing color**

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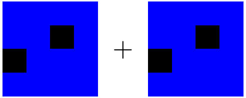
We need **3 B** pixels, but random shares are  $\Longrightarrow$ No way, **hiding failure**

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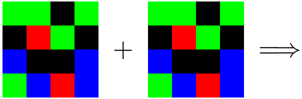
We need **4 R** pixels, but random shares are  No way, **missing color**

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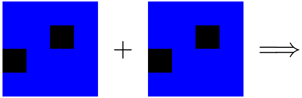
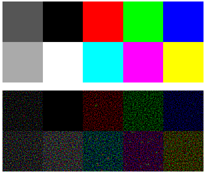
We need **3 B** pixels, but random shares are  No way, **hiding failure**

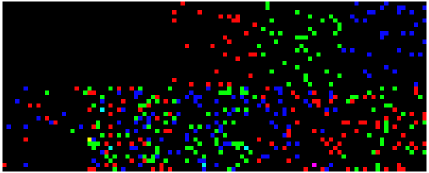
In the ***unhiding*** method, where all pixels are initially hidden, and while swapping the unhiding of unwanted color is not allowed, the hiding failure error cannot appear

Hiding method: Two types of errors

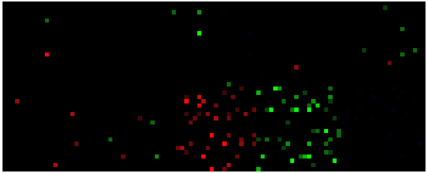
1/2: `test100`



decoded



missing color errors

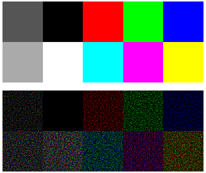


hiding failure errors

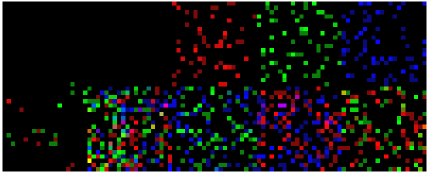
- Note the relation of error density to color: more white more errors
- Number of *hiding failures* is generally smaller than that of *missing colors*

Unhiding method: One type of errors

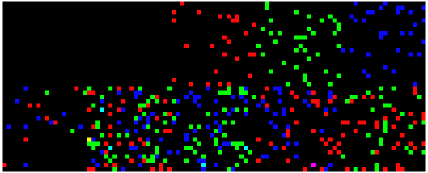
1/2: `test100`



decoded



missing color errors

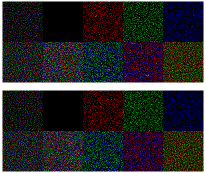


For comparison: *missing color* from *hiding*

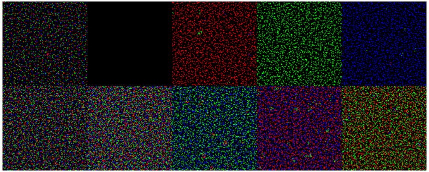
- Note the relation of error density to color: more white more errors
- Number of *hiding failures* is generally larger than that in the *hiding* method

Two methods: restored images

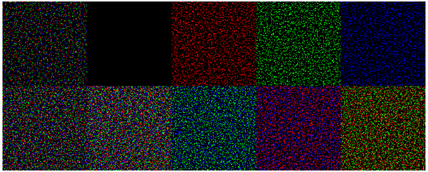
1/2: `test100`



decoded: ↑ *bas* ↓ *no hid*



decoded from *hiding*

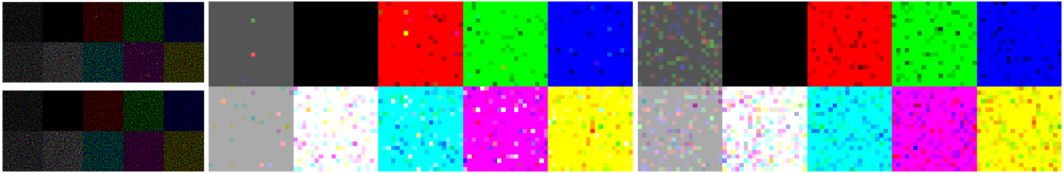


decoded from *unhiding*

- In the *hiding* method pixels can be too bright, there are spikes
- In the *unhiding* method pixels can be too dark, there is granularity

Two methods: restored images

1/2: `test100`



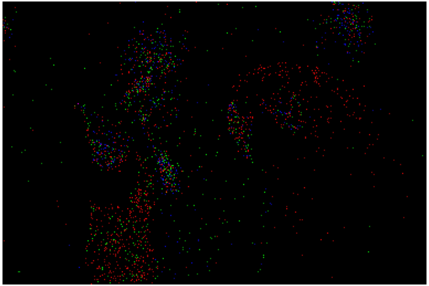
decoded: $\uparrow$ *bas* $\downarrow$ *no hid* restored from *hiding* restored from *unhiding*

- In the *hiding* method pixels can be too bright, there are spikes
- In the *unhiding* method pixels can be too dark, there is granularity

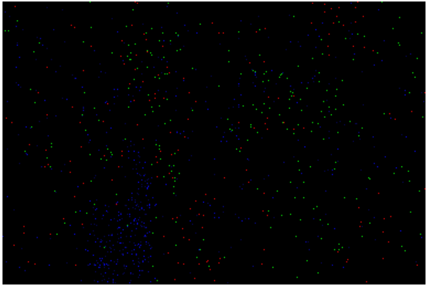
Hidig method: Two types of errors

2/2: `parrots`

decoded



missing color errors



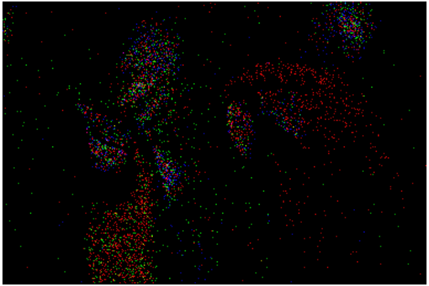
hiding failure errors

- Note the relation of error density to color: more white more errors
- Number of *hiding failures* is generally smaller than that of *missing colors*

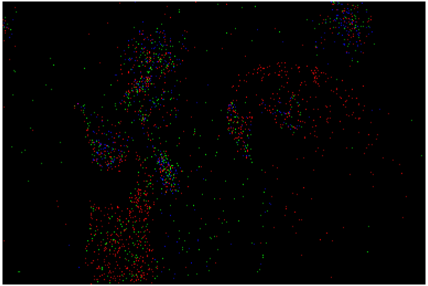
Unhiding method: One type of errors

2/2: parrots

decoded



missing color errors



For comparison: *missing color* from *hiding*

- Note the relation of error density to color: more white more errors
- Number of *hiding failures* is generally larger than that in the *hiding* method

Two methods: restored images

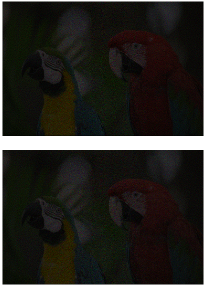
2/2: `parrots`

decoded: ↑ *bas* ↓ *no hid* | decoded from *hiding* | decoded from *unhiding*

- In *hiding* method there are spikes
- In *unhiding* there is granularity

Two methods: restored images

2/2: `parrots`



decoded: $\uparrow$ *bas* $\downarrow$ *no hid*

restored from *hiding*

restored from *unhiding*

- In *hiding* method there are spikes
- In *unhiding* there is granularity

Testing the randomness

Simulation of a series of experiments

- We used the NIST randomness tests (Bassham, Rukhin, Soto, Nechvatal, Smid, Leigh, Levenson, Vangel, Heckert, and Banks 2010): a battery of 15 advanced tests, some with many subtests (188 tests together).
- Tests were performed for six known test images (`parrots`, `peppers`, `Lena`, ...). For each, 100 realizations of coding were simulated.
  - both shares were analyzed $\longrightarrow$ 2 cases,
  - pixels were read by rows and by columns $\longrightarrow \times 2$ cases $= 4$ cases per image,
  - pixels in R, G, B, K were represented by 00, 01, 10, 11 (arbitrary choice).
- p-values for each of 4×100 realizations $\times 188$ tests, for benchmark images, were recorded
- These 75 200 data per image were presented in a compact form, in one page of graphs (Chmielewski, Nieniewski, and Orłowski 2022a)
- The $6 \times 188 \times 4 = 4512$ histograms (6 images, 188 tests, 2 shares, 2 directions) were tested for representing a random process
- Qn: Did the modification of share 2 introduced loss of randomness?

Source: (Chmielewski, Nieniewski, and Orłowski 2021b), probably the first extensive randomness tests.

Statistical evidence for randomness

- In any test, the hypothesis of randomness can be rejected.
  But randomness cannot be proven.
  Hence, the more tests the better, but never too many.
- We have already used the results of NIST randomness tests to analyze *p*-values and to show graphical evidence of randomness (Chmielewski, Nieniewski, and Orłowski 2022a). Now we shall test it statistically.
- We analyze these 4512 sets of *p*-value vectors, 100 elements each, in two ways:
  - by counting how many fall below a fixed rejection threshold ($\alpha = 0.01$),
  - by testing their histogram for deviations from uniformity using the Kolmogorov–Smirnov and chi-squared tests. This second-order statistical analysis enables a more rigorous evaluation of the method's ability to preserve randomness.
- **Shortly: a sequence of bits is random, if the histogram of *p*-values in a test is flat.**
  We shall spare you the trouble of looking at all the histograms.

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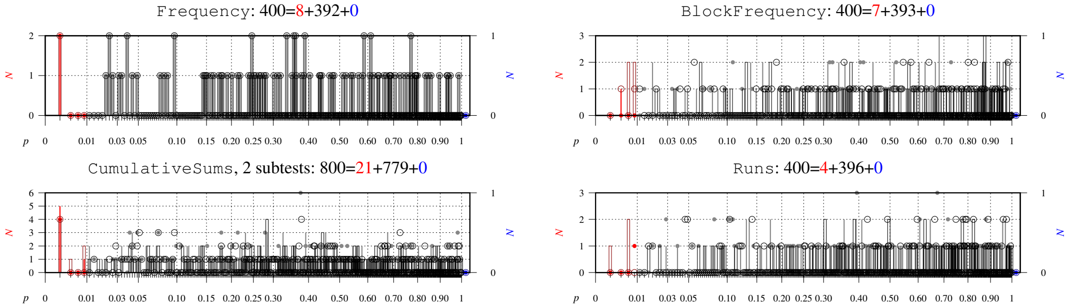
Histograms of p-values for parrots



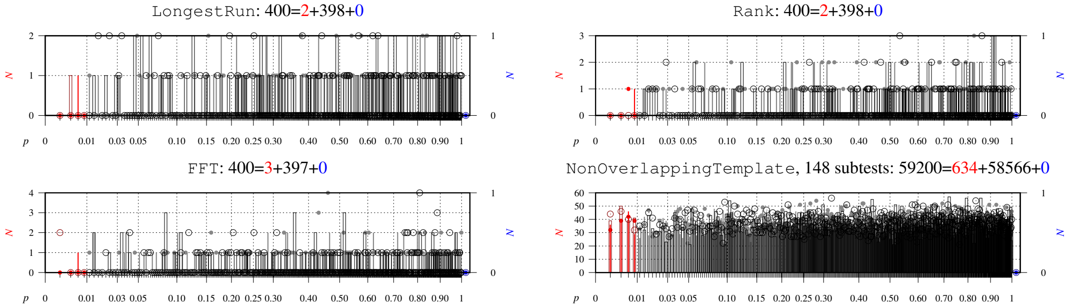
1/4

- Histograms are flat and exhibit expected numbers of failures ($p \leq \alpha, \alpha = 0.01$)
- No reason to reject the hypothesis of randomness, so success

key for graphs

Source: (Chmielewski, Nieniewski, and Orłowski 2021a).

Histograms of *p*-values for parrots



2/4

- Histograms are flat and exhibit expected numbers of failures ($p \leq \alpha, \alpha = 0.01$)
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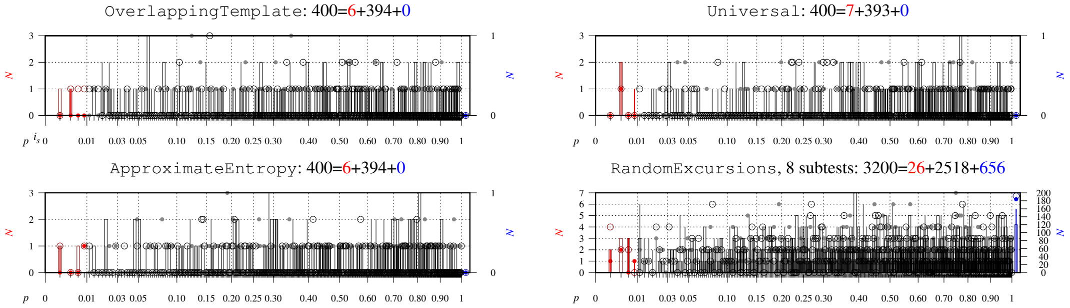
Source: (Chmielewski, Nieniewski, and Orłowski 2021a).

Histograms of p-values for parrots



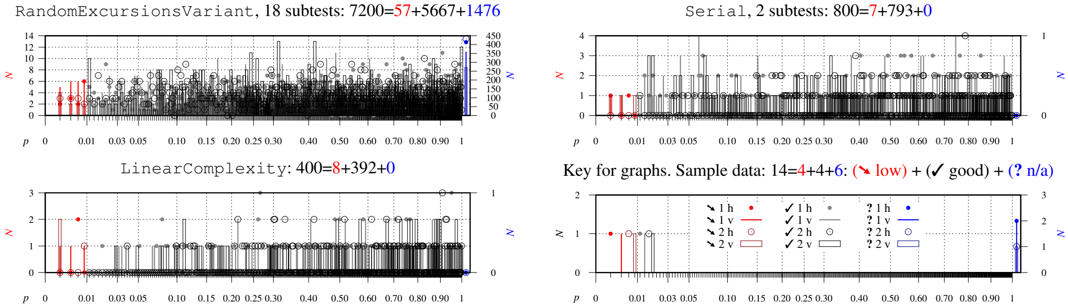
3/4

- Histograms are flat and exhibit expected numbers of failures ($p \leq \alpha, \alpha = 0.01$)
- No reason to reject the hypothesis of randomness, so success

▸ key for graphs

Source: (Chmielewski, Nieniewski, and Orłowski 2021a).

Histograms of *p*-values for parrots

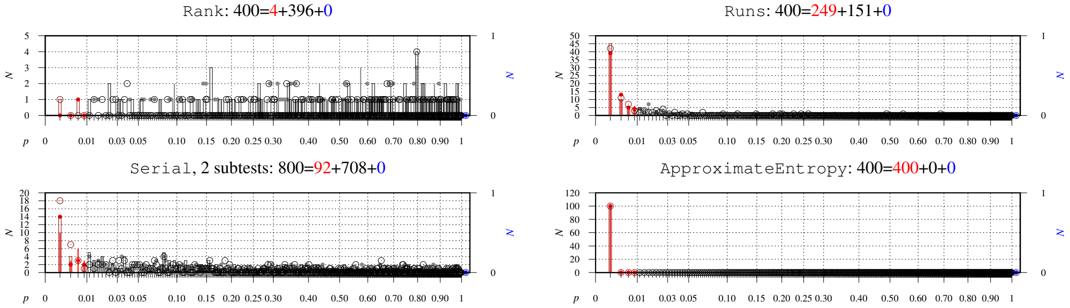


4/4

- Histograms are flat and exhibit expected numbers of failures ($p \leq \alpha, \alpha = 0.01$)
- No reason to reject the hypothesis of randomness, so success

Source: (Chmielewski, Nieniewski, and Orłowski 2021a).

Negative examples for one of previous methods, for parrots



To see ↑ how the negative evidence looks like

- Some histograms are flat, but some are clearly not; too many failures (red)
- There are reasons to reject the hypothesis of randomness, so failure

key for graphs

Source: (Chmielewski, Nieniewski, and Orłowski 2021c).

By test

Table: Rejections by Test

Test	Share 1	Share 2	Rows	Cols	Fails/Samps
Hiding method					
p-v	43	36	45	34	79/4512
K-S	16	29	16	29	46/4512
χ^2	19	27	23	23	46/4512
Unhiding method					
p-v	49	51	49	51	100/4512
K-S	16	20	21	15	36/4512
χ^2	24	25	28	21	49/4512

p-v – counting small *p*-values; K-S – Kolmogorov–Smirnov test; χ^2 – chi-squared test

By share/direction

Table: Rejections by Shares/Directions

Case	Counts/4512 Samples		
Hiding method			
By shares	Share 1: 69	Share 2: 73	Both: 8
By directions	Rows: 74	Columns: 68	Both: 6
Unhiding method			
By shares	Share 1: 77	Share 2: 84	Both: 7
By directions	Rows: 83	Columns: 78	Both: 9

See the paper for more comparisons.

Discussion

- Numbers of rejections were small with respect to the number of samples – close to 1%.
- Rejections in the modified share 2 were not evidently larger, and in some cases smaller than those in the readily generated share 1.
- The cases of rejecting the randomness by all three test or pairs of tests were not a rule.
- Evidence for non-randomness of color distributions in both shares is similarly small.
- NIST tests detected rejections of randomness with very different frequency.
  - `OverlappingTemplate` detected over ten rejections in both methods – *hiding* and *unhiding*.
  - A number of tests detected from 1 to 3 rejections.
  - A number of tests appeared not to detect any lack of randomness in any sample, in at least one method: `ApproximateEntropy`, `CumulativeSums 2`, `LinearComplexity`, `Rank`, `Runs`, `Universal`, and some subtests of `NonOverlappingTemplate`, `RandomExcursions` and `RandomExcursionsVariant`.

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Conclusion

- We analyzed whether the generation of the second share in visual cryptography compromises its statistical randomness.
- We found no evidence of such degradation.
- Standard randomness tests and second-order evaluations of p-value distributions were used.
- Our method for constructing the second share produces outputs that remain statistically indistinguishable from noise.
- This confirms its suitability for secure visual encryption.
- This also demonstrates that controlled determinism can coexist with apparent randomness.
- More broadly, this shows that meta-analysis of test outputs provides a powerful tool for validating the integrity of cryptographic structures under transformation.

Thank you

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Key for histograms of *p*-values

Key for graphs. Sample data: 14=4+4+6: (↘ low) + (✓ good) + (? n/a)