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Combined Change Detector based on Competitive Filters and Statistical Tests

Leszek J. Chmielewski
Department of Informatics
Warsaw University of Life Sciences
Warsaw, Poland
leszek_chmielewski@sggw.pl

Konrad Furmańczyk
Department of Applied Mathematics
Warsaw University of Life Sciences
Warsaw, Poland
konrad_furmanczyk@sggw.pl

Arkadiusz Orłowski
Department of Informatics
Warsaw University of Life Sciences
Warsaw, Poland
arkadiusz_orlowski@sggw.pl

ABSTRACT

A method for detection of changes in one-dimensional data based on a successful combination of a detector consisting of competitive filters and statistical tests is elaborated. Using synthetic data, without noise and with noise of various strengths, our method is compared with the V-Box method and the CUSUM chart. The efficiency of the algorithms is evaluated with the True Positive Rate and the False Positive Rate. The performance of the proposed change detector is demonstrated to be reasonable. The presented method can be applied to any kind of one-dimensional data, especially to time series.

CCS CONCEPTS

• **Computing methodologies** → *Object detection*; *Object identification*; • **Mathematics of computing** → *Probability and statistics*;

KEYWORDS

competitive filter, change detector, statistical methods, experimental verification

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1 INTRODUCTION

The concept of change detection in a one-dimensional signal to be investigated here has its origin in the idea of a filter designed for images named the *competitive filter* [15, 16]. It was noticed in [4] that the idea of two filters working at two opposite sides of an edge in an image could be used to design an edge detector. In two-dimensional signals this concept was discontinued [5] but for one-dimensional signals its various versions were further developed. The concept was presented as viable in detection of changes and complemented with some degree of robustness in [7], the Hough transform-like robustness was introduced in [8], and in [6] a statistical test was used to reduce the false positive detections.

Surveys of various aspects of the change-point problem and related procedures can be found in [1, 3, 12, 18]. The frequently used change-point detection schemes are CUSUM procedure [22] and EWMA control chart [13]. An interesting approach is presented in [20] where a nonparametric detection rule relies on the concept of a moving vertically trimmed box. An overview of the change-point problem can be found in [10]. In this paper the notions of change point, jump point, and discontinuity will be used interchangeably.

In the present paper we shall follow the direction set up in [6]. The combined competitive and statistical detector will be tested on noisy data to see where are its limits. It will also be compared with two frequently used detectors of changes known from the literature.

The filtering function of the considered method was treated as of less importance than the change detection function. The problem of detecting changes in signals is strongly represented in the literature: see for example the surveys [2, 14] for the domain of image processing and [11, 21] for change detection. The concept of competitiveness as stated in our papers cited above seems not to have been explored elsewhere.

In the competitive detection no assumption on the nature of the data is made. The data are approximated with two polynomial functions, one on each side of the considered point, and in the cases used in this paper these will be only the affine functions. In the considered point there can be a discontinuity in the data, and the strength of this discontinuity is measured to detect a change. An assumption that the noise can be represented with the Gaussian distribution is made in the statistical test of the significance of the detected change. These simplifying assumptions are not intrinsic with respect to the method itself and can be developed into a more advanced form.

This paper is organized as follows. In the next Section the general concept of the competitive detector will be very briefly reminded, together with the heuristic as well as the statistical criteria of the existence of a jump. In Section 3 the detectors to be used as benchmark methods will be presented. The test data will be described in Section 4. The results received will be presented in Section 5 and discussed in Section 6. In the last Section the paper will be concluded and the directions for future work will be outlined.

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2 THE METHOD

2.1 General Concept

The following short description will be an abbreviation and simplification of the broader explanation made in [6]. A sequence of measurements $z(x) = y(x) + n(x)$, with discrete independent variable x and noise $n(x)$, will be considered. The filtering and detection is performed in the *central point* x_0. It is assumed that the measurements are known at both sides of x_0, in the closed interval $[x_0 - s - 1, x_0 + s + 1]$, where s is the *scale* of the detector. The detector is *competitive* in that two approximators, that is, the *Left* and the *Right* one, are used to find $y(x_0)$. The first one uses $z(x), x \in [x_0 - s - 1, x_0 - 1]$ to find $\hat{y}_L(x_0)$. The second one uses $z(x), x \in [x_0 + 1, x_0 + s + 1]$ to find $\hat{y}_R(x_0)$. If x is time, it can be said that the left approximator operates in the past, and the right one in the future with respect to x_0.

Errors made by each approximator, $e_L(x_0)$ and $e_R(x_0)$, respectively, can be estimated. As the filtered value at the central point, $\hat{y}(x_0)$, the output of that filter which has a smaller error is taken. This value will be of no interest to us. Here, linear least square approximators are used and their mean square errors are used as the approximation errors [4] (see [7, 8] for other solutions).

2.2 Heuristic Criterion of a Jump

As the chage, the jump E_0 between the function values $z(x_0 - 1), z(x_0 + 1)$, if present, can be found from the results from two approximators

$$E_0(x_0) = \hat{y}_R(x_0 + 1) - \hat{y}_L(x_0 - 1) \tag{1}$$

The question remains, where is the jump.

The conditions for the existence of the jump is that the graphs of the approximation errors cross in such a way that for increasing x the error from the past goes up and for decreasing x the error for the future goes up, which can be expressed as

$$\begin{array}{lcl} e_R(x_0 - \delta) > e_L(x_0 - \delta) & \wedge & e_R(x_0 + \delta) < e_L(x_0 + \delta) \\ e_R(x_0 - \delta) > e_R(x_0 + \delta) & \vee & e_L(x_0 - \delta) < e_L(x_0 + \delta) \end{array} \tag{2}$$

The crossing of the graphs of errors around a jump is illustrated in Fig. 1, where data with no noise are shown for clarity. The process of filtering and edge detection can be imagined so that the central point, with the two approximators at its both sides, move along the data from left to right. When a step is encountered, first the right approximator moves over it. The step enters the right approximator's support. Therefore, the error of the right approximator goes up, as in Fig. 1a. As the central point moves forward, the step leaves the support of the right approximator, so its error goes down, and it reaches a minimum when the approximators surround the jump (zero in this ideal case), as in Fig. 1b. Further, the jump enters the support of the left approximator, as in Fig. 1c, so now the error of the left approximator increases. Hence, the graphs of errors cross over x in which the step is located. When both approximators move on, their errors decrease.

An additional, self-explaining condition for the step is that its value $E_0(x_0)$ is larger than a specified threshold.

This detector will be further referred to as the heuristic competitive detector and denoted C.

Two remarks should be made here. The first remark is that the detector finds the jump at both data points between which the jump occurs. This is the consequence of symmetry between the past and the future in the design of the detector, which does not hold for the classical change detectors. The second remark is that the presented concept works well only if the distance between consecutive jump points is not smaller than the scale of the detector; otherwise the scheme presented in Fig. 1 fails.

The the software for the competitive detector as well as Fig. 1 were created with MATLAB® [17].

2.3 Statistical Criterion of Change Significance

Crossing the graphs of errors described in the previous Section goes on precisely in the described way provided that the edges are isolated, with respect to the scale s. However, it is not always so. Sometimes the false positive detections as well as false negative ones can occur. This is why we have introduced a simple mechanism of additionally testing the edge significance in a statistical way, to exclude false positive detections.

Let us assume that the sequence of measurements form a piecewise linear signal with additive Gaussian noise. For an isolated point x_0 it is observed $y(x) = a_L + b_L x + \epsilon_x$ for $x < x_0$ and $y(x) = a_R + b_R x + \epsilon_x$ for $x \geq x_0$, where the noise ϵ_x has a zero mean normal distribution. There is a jump at x_0 if $\theta = a_R - a_L \neq 0$. Let us verify a hypothesis $H_0 : \theta = 0$ – the jump is absent, against the alternative hypothesis $H_1 : \theta \neq 0$ – the jump is present. To verify this, the test statistics $|\hat{y}^s_R(x_0) - \hat{y}^s_L(x_0)|$ is used, where $\hat{y}^s_L(x_0)$ is a linear regression function of s points on the left of x_0, without this point, that is, from the set $X_L = x \in [x_0 - s, x_0 - 1]$, and $\hat{y}^s_R(x_0)$ is a linear regression function of s points on the right of x_0, with this point, that is, from the set $X_R = x \in [x_0, x_0 + s - 1]$. An isolated jump is detected if

$$P(|\hat{y}^s_R(x_0) - \hat{y}^s_L(x_0)| > t_\alpha) = \alpha\,, \tag{3}$$

where α – significance level (value $\alpha = 0.05$ will be assumed throughout) and t_α is a critical value that corresponds to α. Provided the hypothesis H_0 holds, the distribution of $\hat{y}^s_R(x_0) - \hat{y}^s_L(x_0)$ is zero-mean normal with variance $\sigma^2_L + \sigma^2_R$, where $\sigma^2_L = \mathrm{Var}(\hat{y}^s_L)$ and $\sigma^2_R = \mathrm{Var}(\hat{y}^s_R)$.

Let Φ be a cumulative distribution function of the standard normal distribution. Therefore,

$$t_\alpha = \sigma \Phi^{-1}(1 - \alpha/2)\,, \tag{4}$$

where $\sigma = \sqrt{\sigma^2_L + \sigma^2_R}$. Detailed information can be found in [6].

In our considerations we use the Gaussian noise model as a way to interpret the data in which unknown processes give rise to complex patterns and in which we seek an explanation in terms of simplified events.

This detector will be further referred to as the statistical detector and denoted S.

2.4 Results for Both Criteria

In the heuristic detector, as the jump in x_0 the relation between measurements concerning points $x_0 - 1, x_0 + 1$ are considered, while in the statistical detector this concerns $x_0 - 1, x_0$. To use the condition (3) in the common setting together with the conditions (2)

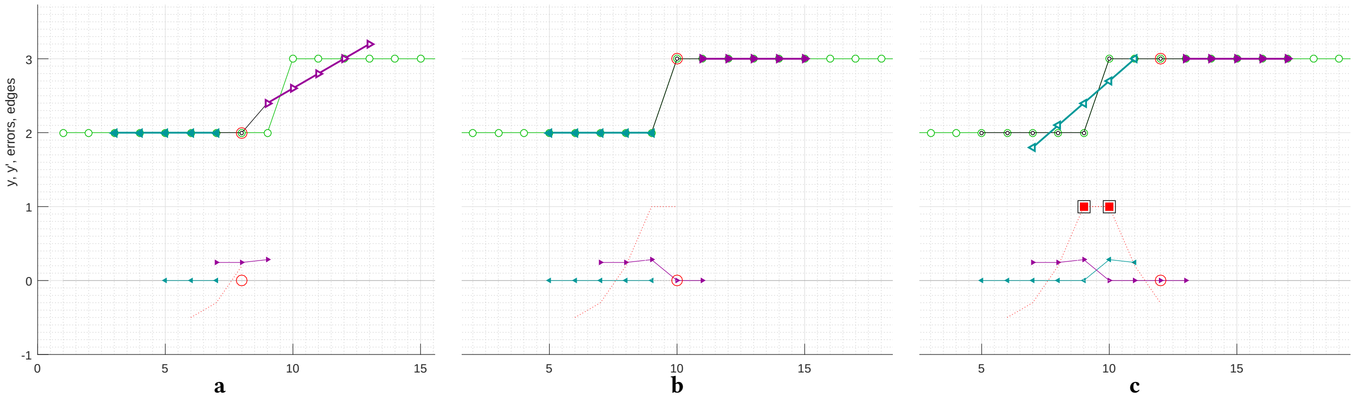

Figure 1: Intermediate results for the two approximators, (a) for $x_0 = 8$, in front of the jump, (b) for $x = 10$, at the jump, and (x) for $x_0 = 12$, behind the jump. Dotted red line is the graph of jump according to (1). Thin magenta and cyan lines are graphs of errors. They cross between points 10 and 11. Pale green (■): function; dark cyan (■): left error, dark magenta (■): right error; red square (■): point where the function has a jump (black square indicates that S detector confirms the jump). Current central point x_0 marked with a red circle. The left and right approximators around x_0 shown with thicker cyan and magenta lines. The approximator with zero error has full triangular marks, the other one has marks filled with white.

the following should be noted. Due to the structure of sets X_L, X_R around x_0, the following relations between the approximations from the heuristic notation and the regressions from the statistical notation hold

$$\begin{aligned} \hat{y}_L^s(x_0) &= \hat{y}_L(x_0), \\ \hat{y}_R^s(x_0) &= \hat{y}_L(x_0-1), \end{aligned} \tag{5}$$

and similarly the error measures from the heuristic notation can be related to the standard residual errors from the statistical notation.

To come to a common meaning of a jump in x_0 it can be considered, in the statistical formulation, that a jump exists if there is a jump between $x_0 - 1, x_0$ or between $x_0, x_0 + 1$. If only one jump exists, its value θ is taken as the step of the function $y(x)$. If both are present, the one having a larger modulus is taken.

If there is an edge according to (2) and (3), then a statistically significant edge exists. If (2) holds and (3) does not, than the edge is statistically insignificant and it is dismissed. If (2) is false, then there is no need to check (3), although here both conditions are calculated independently to show the results in a detailed way.

This detector will be further referred to as the combined heuristic and statistical detector and denoted CS.

3 BENCHMARK DETECTORS OF CHANGE

As the benchmark detectors we use the parametric Vertical Box Control Chart (V-Box) [9] and CUSUM chart [10, 22].

The implementation of the V-Box method merely requires counting the number of data points which fall into the box attached to the last available observation. More formally, the algorithm can be defined as follows. At the current time $N+1$ the value of the statistic

$$T = \sum_{i=1}^{N} \mathbf{1}\,(z(N+1) - H \leq z(i) \leq z(N+1) + H) \tag{6}$$

is calculated, where $\mathbf{1}$ is the indicator function. If the value is greater than γN, then N is replaced by $N+1$ and this step is repeated. Otherwise the algorithm stops and $q = N+1$ is output, where q is the jump point. The parameters $\delta, H > 0$ and $\gamma \in (0,1)$ are fixed. In [9] the optimization criterion for H and γ is given.

The CUSUM chart is defined as the CUSUM statistics $C_i = \max(0, C_{i-1} + z(i) - k)$, where k represents the reference value. An alarm is triggered when C_i reaches the control value h, which is determined to maintain a given in-control average run length (ARL) before a false out-of-control alarm is raised.

4 TEST DATA

We consider the following as the model change in the data

$$z(x) = \begin{cases} n(x) & \text{for } x = 1, 2, \ldots, 20, \\ b + n(x) & \text{for } x = 21, 22, \ldots, 40, \end{cases}$$

where $n(x)$ is i.i.d. random noise with Gaussian distribution with zero mean and variance σ^2. In our model the jump appears at $x = 21$ with $b = 1$, so this is a unit jump. We replicate this model 1000 times. We work with data sets with different Signal to Noise Ratio, which is defined as $\text{SNR} = 10\log_{10}(b^2/\sigma^2)$ [dB]. The values of σ for noise levels which will be used further are shown in Table 1. Fragments of our simulated data sets are depicted in Figure 2.

5 RESULTS

The detectors proposed in this paper have been compared with the parametric V-Box method [9] for two scenarios: $\delta = 0.2$ and $\delta = 0.5$ and with $\epsilon = 1$ in both cases, and with CUSUM chart [22].

For applying the V-Box method we chose $N = 20$ points to create the V-Box and in an essential order we searched for a jump outside the V-Box. The data were divided by known σ and optimal

Table 1: Parameter σ of the Gaussian noise for SNR values

SNR	6	9	12	15	18	24	∞
σ	0.501	0.355	0.251	0.178	0.126	0.063	0

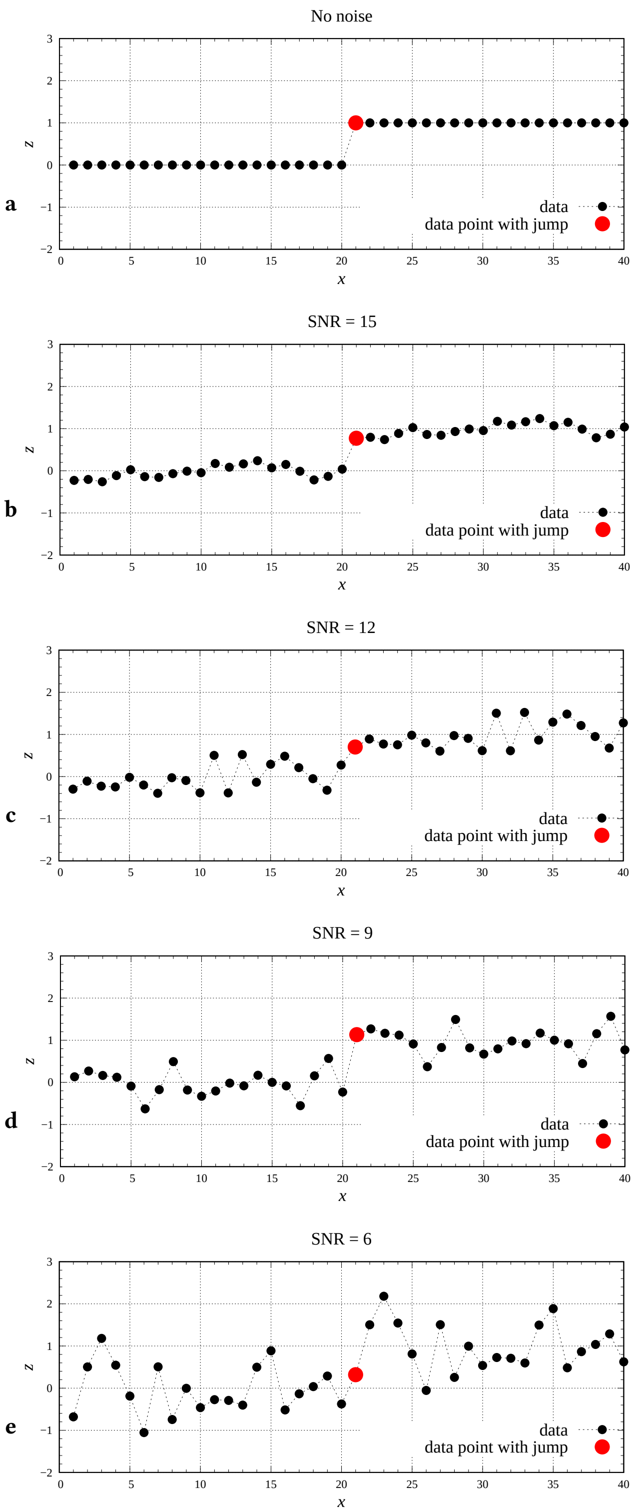

Figure 2: Fragments of data sets for selected noise levels. Jump points marked in red. Dotted lines between data points have no significance other than to indicate the sequence of data, in this Figure as well as in Figure 3.

Table 2: TPR and FPR for V-Box for $N = 20$, $\delta = 0.2$. In Tables 2-8 the values considered as acceptable are typeset in bold (see also text).

SNR	6	9	12	15	18	24
TPR	0.560	**0.814**	**0.979**	**0.999**	**1**	**1**
FPR	0.263	0.428	0.612	0.645	0.648	0.646

Table 3: TPR and FPR for V-Box for $N = 20$, $\delta = 0.5$

SNR	6	9	12	15	18	24
TPR	**0.826**	**0.952**	**0.998**	**1**	**1**	**1**
FPR	0.682	0.823	0.869	0.866	0.867	0.865

Table 4: TPR and FPR for V-Box for heuristic reasoning

SNR	6	9	**12**	**15**	**18**	**24**
TPR	0.355	0.647	**0.929**	**0.998**	**1**	**1**
FPR	**0.071**	**0.103**	**0.155**	**0.190**	**0.189**	**0.189**

Table 5: TPR and FPR for CUSUM

SNR	6	9	12	15	18	24
TPR	0.114	0.222	0.412	0.495	0.500	**1**
FPR	**0.197**	0.276	0.392	0.498	0.504	1

parameters H and γ were chosen for Gaussian model in those scenarios (for details see [9]). For $\delta = 0.2$ we obtained: $H = 1.95$ and $\gamma = 0.5$ and for the case $\delta = 0.5$ we obtained: $H = 1.41$ and $\gamma = 0.6$. We also applied V-Box method for selected parameters $\gamma = 1/20$ and $H = \sigma$ by the heuristic reasoning.

For a CUSUM chart we used Smisc package from R software [19]. We obtained optimal parameters for the CUSUM chart from the CUSUMdesign package in R. Putting $k = 0.5$ and ARL $= 100$ we obtained $h = 2.931$.

The True Positive Rate (TPR) and the False Positive Rate (FPR) of our detectors: heuristic C, statistical S, and C combined with S denoted as CS for the scale s from 3 to 19 are presented in Figures 3a-e.

The results of the detectors proposed by us and the benchmark detectors are presented in Tables 2-8. In these Tables, the values of error measures considered as acceptable are typeset in bold. For the comparisons to be made here we have assumed rather arbitrarily the thresholds or acceptability for TPR and FPR to be 0.8 and 0.2, respectively. Hence, an acceptable TPR should not be less than 0.8 and an acceptable FPR should not exceed 0.2. We shall consider that when at least one of these thresholds are surpassed, the respective detector breaks down. The SNR values for which both TPR and FPR are acceptable in this sense (the detector does not break down) are also typeset in bold.

The results from the V-Box method are given in Tables 2, 3 and 4. The results for CUSUM chart are presented in Table 5. These

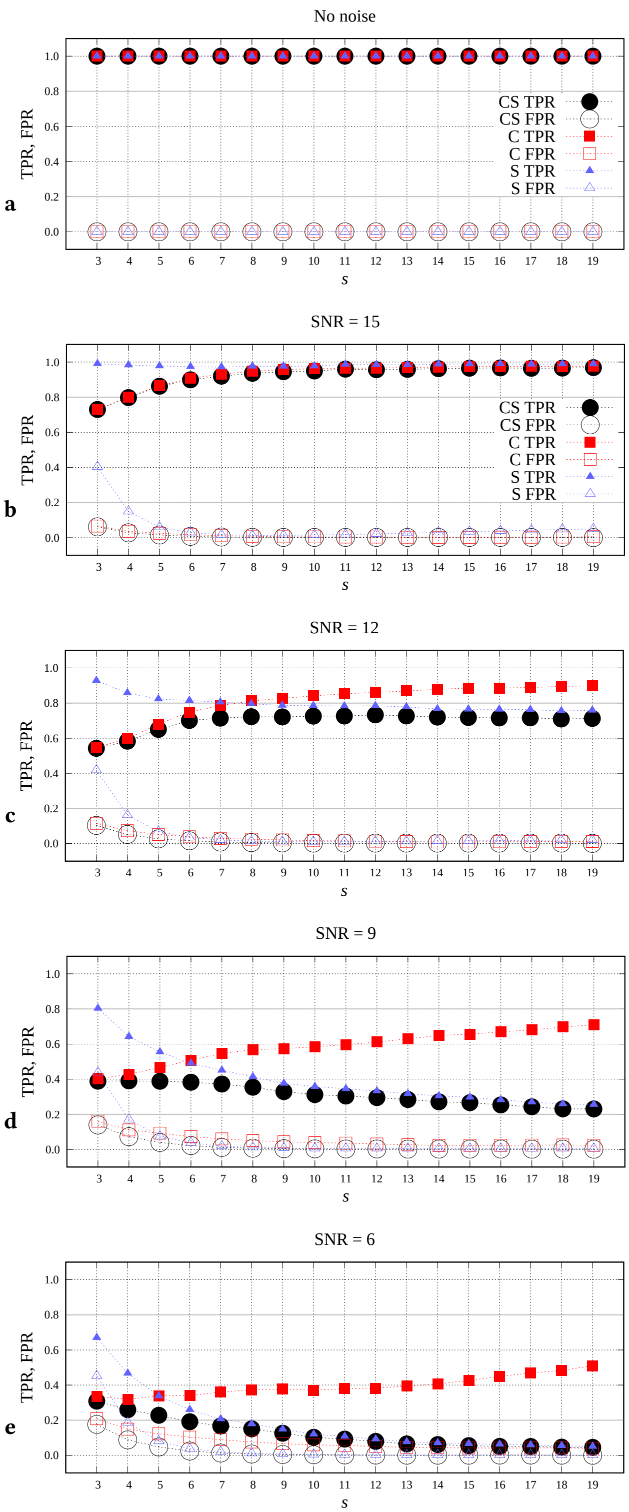

Figure 3: TPR and FPR versus scale for detectors C, S and CS, for selected noise levels. Legend not shown for large values of noise due to lack of room in the graphs. Gray lines indicate acceptability thresholds for TPR and FPR (see text).

Table 6: TPR and FPR for C detector

SNR	6	9	12	15	18	24	∞
TPR	0.510	0.709	**0.898**	**0.997**	**0.997**	**1**	**1**
FPR	**0.041**	**0.024**	**0.012**	**0.006**	**0.001**	**0**	**0**

Table 7: TPR and FPR for S detector

SNR	6	9	12	15	18	24	∞
TPR	0.050	0.256	0.759	**0.989**	**1**	**1**	**1**
FPR	**0.002**	**0.005**	**0.020**	**0.049**	**0.085**	**0.111**	**0**

Table 8: TPR and FPR for CS detector

SNR	6	9	12	15	18	24	∞
TPR	0.045	0.231	0.713	**0.968**	**0.997**	**1**	**1**
FPR	**0.001**	**0.002**	**0.001**	**0.001**	**0.000**	**0**	**0**

detectors do not work without noise, so there are no results for SNR = ∞.

The best results for our detectors are given in Tables 6-8. For SNR from 6 to 12 the best scale s is 19, which is the maximum scale such that the approximators can span over a fragment of data without a jump. For small noise, that is, for SNR = 18 and 21 the best scale is 18, and for SNR = 24 and ∞ the best scale is 17. In any case, the difference of TPR and FPR of our detectors for s from 17 to 19 is negligible.

6 DISCUSSION

For the strongest noise considered here, SNR = 6, we observe that our heuristic detector C has TPR less than the V-Box method for $\delta = 0.2$, but our detectors C, S, and CS have much less FPR than both V-Box method. For SNR = 18 and SNR = 24 our detectors have TPR close to 1 and FPR close to 0, or exactly 0 for SNR = ∞. In those cases the V-Box methods have large FPR. In all the cases CUSUM chart has small TPR and large FPR, which indicate that this method does not work well for the data considered. V-Box method ($H = \sigma$, $\gamma = 1/20$) has the largest TPR, but FPR is also larger than that for C, S, and CS detectors. We can observe that for noise with SNR greater than 6 our detectors C, S, and CS present an improvement over CUSUM and V-Box. In all the cases the detector S has less TPR than the detector C. The use of the statistical test in the CS detector reduced the FPR. For the three competitive detectors C, S, and CS the optimal scale is practically always $s = 19$, which is the largest possible scale still making it possible for the two approximators used in the C and CS methods to span over a data range containing at most one jump simultaneously.

The competitive detector C breaks down (its results surpass the acceptability thresholds defined beforehand) between SNR 12 and 9. The combined competitive and statistical detector CS breaks down for smaller noise, between SNR 15 and 12, but for this detector the FPR is slightly smaller. The best results received with the V-Box detector (Table 4) indicate that it breaks down between SNR 12 and

9, like the C detector; however, the FPR for each of the competitive detectors (C and CS) is much smaller than that for the V-Box.

For this simulation study it can be said that the detectors C, S, and CS are more efficient than the V-Box method.

7 CONCLUSION

We have tested the detectors proposed by us with the use of simulated data sets. The data contained a series of unit jumps, and were without noise, SNR $= \infty$, and with noise characterized by SNR from 24 to 6. In all cases our detectors: heuristic C, statistical S, and combined CS behaved more reasonably than the benchmark detectors V-Box and CUSUM. The presented methods and observations can be applied to any noisy one-dimensional data, especially to time series.

It can be stated that the strength of the competitive detectors lies in that a large number of data points on the left as well as those on the right with respect to the jump point are considered. If a time series is considered, this would mean that the data from the past with respect to the jump point as well as those from the future are considered simultaneously. If the information on the existence of a jump should be obtained as quickly as possible, this is not acceptable. However, in an offline, post-factum analysis, such methods as proposed by us could be applied.

Encouraged by the good performance of the simple heuristic approach used in the design of the competitive detectors we can hypothesize that also in statistically orientated detectors, by increasing the number of data points pertaining to the future with respect to the considered data point could improve their performance. The number of data points pertaining to the past does not have to be equal to that pertaining to the future. These directions of study will be the subject of our future work.

REFERENCES

[1] M. Basseville and I. V. Nikiforov. 1993. *Detection of Abrupt Changes, Theory and Application.* Prentice-Hall, Englewood Cliffs, New Jersey, USA.

[2] S. Bhardwaj and A. Mittal. 2012. A Survey on Various Edge Detector Techniques. *Procedia Technology* 4 (2012), 220–226. https://doi.org/10.1016/j.protcy.2012.05.033

[3] B. E. Brodsky and B. S. Darkhovsky. 2002. *Nonparametric Methods in Change-Point Problems.* Kluwer, Dordrecht, Netherlands.

[4] L. Chmielewski. 1996. The Concept of a Competitive Step and Roof Edge Detector. *Machine Graphics & Vision* 5, 1-2 (1996), 147–156.

[5] L. Chmielewski. 1997. Failure of the 2D Version of the Step and Roof Edge Detector Derived from a Competitive Filter. Report of the Division of Optical and Computer Methods in Mechanics, IFTR PAS.

[6] L. J. Chmielewski, K. Furmańczyk, and A. Orłowski. 2017. Competitive Detector of Changes with a Statistical Test. In *Proc. 10th Int. Conf. Computer Recognition Systems CORES 2017 (Advances in Intelligent Systems and Computing)*, M. Kurzynski et al. (Eds.), Vol. 578. Springer, 2018, Polanica Zdrój, Poland, 433–442. https://doi.org/10.1007/978-3-319-59162-9_45

[7] L. J. Chmielewski and A. Orłowski. 2017. Detecting Changes with the Robust Competitive Detector. In *Proc. 8th Iberian Conference on Pattern Recognition and Image Analysis IbPRIA 2017 (Lecture Notes in Computer Science)*, L. Alexandre, J. Salvador Sánchez, and J. Rodrigues (Eds.), Vol. 10255. Springer, Cham, Faro, Portugal, 353–362. https://doi.org/10.1007/978-3-319-58838-4_39

[8] L. J. Chmielewski and A. Orłowski. 2018. Detecting Changes in Time Sequences with the Competitive Detector. In *Complex Systems: Solutions and Challenges in Economics, Management and Engineering. Dedicated to Professor Jaime Gil Aluja (Studies in Systems, Decision and Control)*, C. Berger-Vachon, A. M. Gil-Lafuente, J. Kacprzyk, Y. Kondratenko, J. M. Merigó, and F. C. Morabito (Eds.), Vol. 125. Springer, Cham, 251–267. https://doi.org/10.1007/978-3-319-69989-9

[9] K. Furmańczyk and S. Jaworski. 2016. Large Parametric Change-point Detection by a V-box Control Chart. *Sequential Analysis* 35, 2 (2016), 254–264. https://doi.org/10.1080/07474946.2016.1165548

[10] F. Gustafsson. 2000. *Adaptive Filtering and Change Detection.* John Wiley & Sons, Chichester, UK.

[11] W. Hu, T. Tan, L. Wang, and S. Maybank. 2004. A Survey on Visual Surveillance of Object Motion and Behaviors. *IEEE Trans. Systems, Man, and Cybernetics, Part C (Applications and Reviews)* 34, 3 (Aug 2004), 334–352. https://doi.org/10.1109/TSMCC.2004.829274

[12] T. L. Lai. 1995. Sequential Changepoint Detection in Quality Control and Dynamical Systems. *Journal of Royal Statistical Society. Series B (Methodological)* 57, 4 (1995), 613–658. https://www.jstor.org/stable/2345934

[13] C. W. Lu and M. R. Reynolds. 1999. EMWA Control Charts for Monitoring the Mean of Autocorrelated Process. *Journal of Quality Technology* 3 (1999), 166–188. https://doi.org/10.1080/00224065.1999.11979913

[14] R. Maini and H. Aggarwal. 2009. Study and Comparison of Various Image Edge Detection Techniques. *Int. Journal of Image Processing* 3, 1 (2009), 1–11. http://www.cscjournals.org/library/manuscriptinfo.php?mc=IJIP-15

[15] M. Niedźwiecki and W. A. Sethares. 1991. New Filtering Algorithms Based on the Concept of Competitive Smoothing. In *Proc. 23rd ISCIE Symp. on Stochastic Systems and its Applications SSS'91.* ISCIE, Osaka, Japan, 129–132.

[16] M. Niedźwiecki and P. Suchomski. 1994. On a New Class of Edge-preserving Filters for Noise Rejection from Images. *Machine Graphics & Vision* 1-2, 3 (1994), 385–392.

[17] Makers of MATLAB and Simulink. 2018. *MATLAB.* The MathWorks, Inc., Natick, MA, USA. https://www.mathworks.com

[18] H. V. Poor and O. Hadjiliadis. 2009. *Quickest Detection.* Cambridge University Press, Cambridge.

[19] R Core Team. 2018. *R: A Language and Environment for Statistical Computing.* R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org

[20] E. Rafajłowicz, M. Pawlak, and A. Steland. 2010. Nonparametric Sequential Change-Point Detection by a Vertically Trimmed Box Metod. *IEEE Trans. Information Theory* 56, 7 (2010), 3621–3634. https://doi.org/10.1109/TIT.2010.2048443

[21] T. D. Räty. 2010. Survey on Contemporary Remote Surveillance Systems for Public Safety. *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)* 40, 5 (Sept 2010), 493–515. https://doi.org/10.1109/TSMCC.2010.2042446

[22] Y. Ritov. 1990. Decision Theoretic Optimality of the CUSUM Procedure. *The Annals of Statistics* 56, 3 (1990), 1464–469. https://www.jstor.org/stable/2242064